Impact of a hard cylinder with flat surface on the elastic layer

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Summary. Here author investigates the stress-strain states of an elastic layer engaged in impact with a hard cylinder with flat surface. The initial contact proceeds by that flat surface. We consider the contact problem with a dynamically changing contact zone. We use the same approach as in [2, 3], which is based on the reduction of the basic dynamical equations of the system stamp-layer to an infinite system of Volterra integral equations of the second kind. This approach sets the stage for the efficient numerical analysis of the problem and for reliably determining the quantitative dynamical and kinematic characteristic describing the collision process as functions of the initial impact velocity and the parameters of the elastic layer.

Key words: impact**,** elastic, layer, plane problem, hard cylinder.

INTRODUCTION

Problems of impact hard and elastic bodies on the deformable bodies remain relevant and researched in various models and formulations. One of the most important direction of such research is to identify the characteristics of destruction incised beam specimens at their destruction at the three-point bending using indenter. Relevant experiments make it possible to identify much-needed mechanical characteristic of the material − destruction toughness related to the stress intensity factor at the top of crack.

Since the process is dynamic and may be accompanied by substantial plastic deformation, its study is complex and multifaceted problem, which requires analysis of the impact on the experienced striker body, dynamic interaction of body and supports, beginning the process of destruction and its development. This subject is very wide and is associated with numerous publications. It were selected for that paper only sufficient minimum of such publications. In [14] was investigated dynamic problem about pipeline with flowing fluid inside with taking into account Coriolis force. In [1…5] were investigated dynamic problems of outside contact pressure to the metal constructions. The nonstationary problems [6…11] of impact interaction of absolute hard flat indenter with incised in the median section of beam specimens in dynamic elastic-plastic formulation are belong to the underlie theme. In paper [12] three dimension quasistatic problem in elastic-plastic formulation corresponding to [1, 3] was solved. It was revealed that stresses significantly different from the stresses obtained from the solution of a similar problem in the dynamic elastic-plastic formulation. In paper [4] it is solution of problem of plane strain state from three-point bending of the beam sample with middle notch. It was taken in account the process of unloading of the material. The plane stress

[5, 6], stain [8] and spatial [10, 11] problems of growing crack simulation were solved. In [5, 10] the crack length was increasing when at the top of crack the maximal stresses were absent. In [6, 8, 11] the crack was growing by generalized local criterion of brittle fracture. Destruction toughness of the material was determined on the base of solutions of plane strain [7] and spatial [1, 9] problems. The approach of studying the dynamic development of cracks in the experimental samples [26 − 29], based on the method of Rayleigh was proposed.

The bulk of publications of study of the strain-stress state of the impact interaction is in elastic formulation. In elastic formulation plane [2, 15] and axisymmetric [16] problems of the impact of hard bodies on the elastic layer were investigated. In [17] it was investigated the effect of nonstationary loading on the front surface of the elastic half-strips. In [23] the problem of flat elastic dynamic interaction of the absolute hard body with homogeneous isotropic elastic half-space at supersonic stage. Here it is assumed that the contact zone can be multiply region. For solving the initial Cauchy problem for a system of quasi-linear differential equations the hybrid methods were developed. The impact of the hard cylinder [2] is interesting as a limiting case of the impact of an elastic shells [13].

In this paper it was used an approach [2, 13, 24], which based on the reduction of initial dynamics equations of the system shell-layer to the infinite system of Volterra integral equations of the second kind. The size of the initial contact zone between cylinder and top surface of the layer is equal to the width of cylinder flat surface.

STATEMENT OF THE PROBLEM

A hard cylindrical body with flat surface moving transversely along a path perpendicular to the surface of an elastic layer $0 \le z \le H$ reaches the surface at the time $t = 0$. Initial contact is made along the plane flat surface which is parallel to axis of the cylinder. We attach a moving cylindrical coordinate frame rOz' to the stamp, its z axis coinciding with the axis of the cylinder, and with the layer we associate a fixed Cartesian coordinate frame xyz.

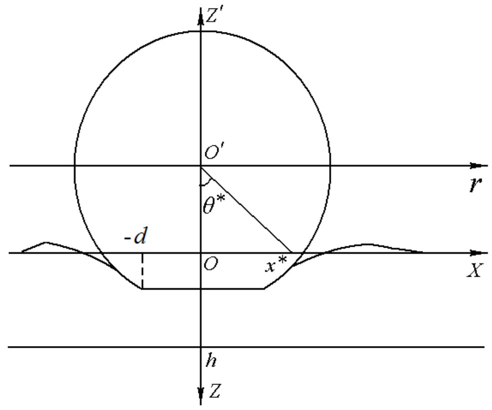

Fig. 1. Schema of system stamp-layer

The stamp penetrates (Fig.1) the elastic layer at a velocity $V_T(t)$, $(0 \le t \le T)$ with the initial value $V_0 = V_T(0)$, where T is the total time of interaction of the stamp with the layer. We introduce the dimensionless variables.

$$t' = \frac{C_0 t}{R},\ x' = \frac{x}{R},\ z' = \frac{z}{R},\ u_t' = \frac{u_t}{R},$$

$$\sigma_{ij}' = \frac{\sigma_{ij}}{K},\ v_T' = \frac{v_T}{C_0},\ w_T' = \frac{w_T}{R},\ p' = \frac{p}{KR},$$

$$q' = \frac{q}{KR},\ M' = \frac{M}{\rho R^2},\ (i, j = x, y, z),$$

$$\beta^2 = \frac{C_S^2}{C_0^2} = \frac{\mu}{K},\ \alpha^2 = \frac{C_p^2}{C_0^2} = \left(1 + \frac{4\mu}{3K}\right),$$

$$C_0^2 = \frac{K}{\rho},\ b^2 = \frac{\beta^2}{\alpha^2} = \frac{3\mu}{3K + 4\mu}.,$$

where ρ, μ, K, C_p and C_s are the density, the shear modulus, the volumetric strain modulus, and the wave propagation velocities in the elastic layer.

The equations of motion of the elastic layer are written in the form [3].

$$\Delta\varphi = \frac{\partial^2\varphi}{\alpha^2\partial t^2},\ \Delta\psi = \frac{\partial^2\psi}{\beta^2\partial t^2},\ \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

If the shear modulus μ is set equal to zero, the equations of motion of the elastic medium go over to the acoustical equations. With Eq.(1) taken into account, the physical quantities are expressed in terms of the wave potentials by the relations

$$u_x = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial z},\ u_x = \frac{\partial\varphi}{\partial z} - \frac{\partial\psi}{\partial x},\ u_y = 0.$$

$$\sigma_{zz} = (1-2b^2)\frac{\partial^2\varphi}{\partial t^2} + 2\beta^2\left(\frac{\partial^2\varphi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}\right),$$

$$\sigma_{xz} = 2\beta^2\frac{\partial^2\varphi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial t^2} - 2\beta^2\frac{\partial^2\psi}{\partial x^2},$$

$$\sigma_{xy} = \sigma_{yz} = 0, \qquad (1)$$

$$\Theta = \sigma_{zz} + \sigma_{xx} = 2(1-b^2)\frac{\partial^2\varphi}{\partial t^2},\ \sigma_{xx} = \Theta - \sigma_{zz},$$

where $u = (u_x, u_y, u_z)$ is the displacements vector, and σ_{zz} and σ_{zx} are the components of the stress tensor. In solving the problem, we use the same approach as in [18, 19, 21], which enables us to identify the linear coordinates along the surface of the layer and the projectile in the early stage of penetration, thereby validating the approximate relations.

$$r \approx \theta,\ \mathrm{ctg}\theta \approx 1/\theta. \qquad (2)$$

According (2) for the displacement u_z and pressure p follow dependents will be performed

$$u_z(t,x,0) = w_T(t) - H(|x|-d)\times$$
$$\times\left(1-\sqrt{1-(|x|-d)^2}\right), \qquad (3)$$

$$w_T(t) = \int_0^t V(\tau)d\tau,\ p(t,x) = -\sigma_{zz}(t,x,0),\ |x| < x^*.$$

Linearized boundary conditions are

$$\left.\frac{\partial u_z}{\partial t}\right|_{z=0} \equiv V(t,x) = v_T(t),\ |x| < x^*(t). \qquad (4)$$

$$\sigma_{zz}\big|_{z=0} = 0,\ |x| > x^*(t),$$
$$\sigma_{zx}\big|_{z=0} = 0,\ |x| < \infty. \qquad (5)$$

At the boundary layer $z = h$ there is hard grip condition.

For interaction time $0 \le t \le T$ from the band we separate a finite size rectangle $\{|x| \le l,\ 0 \le z \le h\}$, whereupon we can treat the problem of impact on the layer as the problem of impact on a strip. The length of the rectangle l so that the disturbances will not reach its lateral boundaries:

$$|x| = l\ \left(l > \alpha(T-t_0) + x^*(t_0),\ \left.\frac{dx^*}{dt}\right|_{t=t_0}\right).$$

We assign zero-valued initial conditions to the problem and on the lateral surface there are sliding fixing conditions

$$u_x\big|_{|x|=l} = 0,\ \sigma_{zx}\big|_{|x|=l} = 0,$$
$$\varphi\big|_{t=0} = \left.\frac{\partial\varphi}{\partial t}\right|_{t=0} = 0,\ \psi\big|_{t=0} = \left.\frac{\partial\psi}{\partial t}\right|_{t=0} = 0. \qquad (6)$$

The movement of the cylinder as body has determined from Newton's second law

$$M\frac{\partial^2 w_T}{\partial t^2} = -F(t),\ V_T(0) = V_0,\ w_T(0) = 0, \qquad (7)$$

where $F(t)$ – reaction force of the elastic layer, which is determined taking into account (3), (5) as an integral of pressure on the contact area:

$$F(t) = 2\int_0^{x^*(t)} p(t,x)dx.$$

Taking into account the elevation of the medium and the slowing of the cylinder penetration into the elastic medium, we determine the boundary of the contact zone x^* from the condition:

$$w_T(t) - u_z(t, x^*, 0) - H(|x^*| - d) \times$$
$$\times\left(1 - \sqrt{1 - (|x^*| - d)^2}\right) =$$
$$\{0, |x| \le x^*(t), \quad \varepsilon < 0, \ |x| > x^*(t)\}.$$

SHEMA AND METHODS

Taking the Laplace transform of Eqs. (2) with respect to the variable t, where s is the parameter of the transform, and applying Fourier separation of variables, we write the general solution of the equations, subject to the conditions of extinction of the disturbances at infinity, in the form [2, 24]

$$\varphi^L(s, x, z) = \sum_{n=0}^{\infty} A_n(s) \times$$
$$\times \exp\left(-z\sqrt{s^2/\text{б}^2 + \text{л}_n^2}\right)\cos \text{л}_n x +$$
$$+\sum_{n=0}^{\infty} B_n(s)\exp\left(z\sqrt{s^2/\text{б}^2 + \text{л}_n^2}\right)\cos \text{л}_n x,$$
$$\psi^L(s, x, z) = \sum_{n=0}^{\infty} C_n(s) \times$$
$$\times \exp\left(-z\sqrt{s^2/\text{в}^2 + \text{л}_n^2}\right)\sin \text{л}_n x + \qquad (8)$$
$$+\sum_{n=0}^{\infty} D_n(s)\exp\left(z\sqrt{s^2/\text{в}^2 + \text{л}_n^2}\right)\sin \text{л}_n x.$$

where $\lambda_n = n\pi/l$, $n = \overline{0,\infty}$ denotes the Eigen values of the problem, determined from the conditions on the lateral surfaces of the half-strip (6).

The functions V, u_z, σ_{zz}, σ_{zx} on the surface of the medium of layer are represented by series in the system of Eigen functions of the problem and the function p is represented by trigonometry series:

$$V(t, x, 0) = \sum_{n=0}^{\infty} V_n(t)\cos\lambda_n x,$$
$$u_z(t, x, 0) = \sum_{n=0}^{\infty} u_{zn}(t)\cos\lambda_n x,$$
$$\sigma_{zz}(t, x, 0) = \sum_{n=0}^{\infty} \sigma_{zn}(t)\cos\lambda_n x, \qquad (9)$$
$$\sigma_{zx}(t, x, 0) = \sum_{n=0}^{\infty} \sigma_{zxn}(t)\sin\lambda_n x.$$
$$p(t, x) = \sum_{n=0}^{\infty} p_n(t)\cos(nx). \qquad (10)$$

In (3) using (9) and making use of the orthogonality of the trigonometric functions the expression for the n-th harmonic of pressure will be:

$$p_n(t) = -\sum_{n=0}^{\infty} \gamma_{mn}(x^*)\sigma_{zz,m}(t),$$
$$\gamma_{mn}(x^*) = \frac{\alpha}{\overline{N}_n^2}\int_0^{\theta^*} \cos nx \cos\lambda_m x\, dx,$$
$$\overline{N}_n^2 = \int_0^{\pi} \cos^2 nx\, dx. \qquad (11)$$

Next the problem for equations (1) with follow boundary conditions has solved.

$$\left.\frac{\partial u_z}{\partial t}\right|_{z=0} = V(t, x), \ \left.\sigma_{zx}\right|_{z=0} = 0, \qquad (12)$$
$$\left.u_z\right|_{z=h} = 0, \ \left.u_x\right|_{z=h} = 0.$$

Satisfying conditions (5) with allowance for (8) and (9), applying the inverse Laplace transform, and invoking the convolution theorem, an equation that establishes an interrelationship between the components of the vertical part of the velocity and the normal stresses on the surface of the layer has been obtained:

$$\sigma_{zn}(t) = -\alpha\left(V_n(t) + \int_0^t V_n(\tau)F_n(t-\tau)d\tau\right), \quad (13)$$

where

$$F_n(t) = \tilde{F}_n(t) + \phi_1(n,t) + \int_0^t (J_0(\beta\lambda_n\xi)\phi_2(n,t-\xi) +$$
$$+ J_0(\text{бл}_n\text{о})\text{ц}_3(n,t-\text{о}))d\text{о}.$$

$$\tilde{F}_n(t) = -\alpha\lambda_n J_1(\alpha\lambda_n t) + 2b\beta\{\beta^2\lambda_n^2 t^2(\bar{J}_0(\alpha\lambda_n t) -$$
$$-\bar{J}_0(\text{вл}_n t) - J_1(\text{бл}_n t) + J_1(\text{вл}_n t)) + \text{вл}_n t(bJ_0(\text{бл}_n t) -$$
$$- J_0(\text{вл}_n t)) + (2-b^2)\bar{J}_0(\text{бл}_n t) - \bar{J}_0(\text{вл}_n t)\},$$

$$\phi_j(n,t) = \gamma_j(n,\alpha,\beta,h) + \sum_{i=2}^{4} H_j(n,s_i,h)\cos\beta_i t,$$

$$\beta_i = |\mathrm{Im}\, s_i|,\ (i=2,3,4),\ (j=1,2,3),$$

$$H_j(n,s_i,h) = 2N_j(n,s_i)/\Delta(s_i), \gamma_j(n,\alpha,\beta,h) =$$
$$= -(\text{д}_{3j}b_0 + B_j/a_1^2 + D_j/b_1^2 + F_j/C_1^2)/a_0,$$

$$\Delta(s) = a_0 s^2(9s^6 + 7(a_1+b_1+c_1)s^4 + 5(a_1b_1 +$$
$$+ a_1c_1 + b_1c_1)s^2 + 3a_1b_1c_1),\ a_0 = b^4h^6/108,$$

$$a_1 = \beta^2\lambda_n^2 + 6\alpha^2/h^2,\ b_1 = \beta^2(\lambda_n^2 + 6/h^2),$$

$$c_1 = 3\beta^2/h^2 + \alpha^2\lambda_n^2,\ N_k(n,s) = a_{0k} + a_{1k}s^2 +$$
$$+ a_{2k}s^4 + a_{3k}s^6,\ (k=1,\ 2),\ N_3(n,s) = a_{03} +$$
$$+ a_{13}s^2 + a_{23}s^4 + a_{33}s^6 + a_{43}s^8, a_{01} = 2h\text{в}^9 b\text{л}_n^2 \times$$
$$\times(1+b^2)(1+h^2\text{л}_n^2 - 2h^2\text{л}_n^4/3), a_{11} = \text{в}^5 b(\text{в}^2 +$$
$$+2h^2\text{л}_n^2(\beta^2(1+b^2)(1+h^2\text{л}_n^2/3) - b^2(2\text{в}^2(1+$$
$$+\text{л}_n^4/3) - h^2\text{л}_n^2(1-b^2)^2/6)))/h,\ a_{21} = \text{в}^5 bh((1+$$
$$+b^2)(1+h^2\text{л}_n^2/3) + h^2\text{л}_n^2(1+2b^2-3b^4)/3))/2,$$

$$a_{31} = \beta^3bh^3(1+6b^2+b^4)/24, a_{02} = 2\beta^{10}b\lambda_n^4 \times$$
$$\times(2b^2 + h^2\text{л}_n^2(2(1+b^2) + 13h^2\text{л}_n^2/20)/3),$$

$$a_{32} = \beta^4b^3\lambda_n^2(1+2b^2+b^4/5)/6,\ a_{12} = 2\beta^8 \times$$
$$\times b\text{л}_n^2(6b^2 + (2+b^2(5+b^2))h^2\text{л}_n^2 + (7/20 + b^2 \times$$
$$\times(7/5+2b^2))h^4\text{л}_n^4)/3, a_{22} = 2\text{в}^6bh^2\text{л}_n^2(b^2(1+$$
$$b^2(10+b^2/5)))h^2\lambda_n^2),\ a_{03} = -2\beta^{10}\lambda_n^4(2+$$
$$+(19/3-b^2)h^4\text{л}_n^4/20 + 2(1-b^2/3)h^2\text{л}_n^2),\ a_{13} =$$
$$-2\text{в}^8\text{л}_n^2(2+(2+b^2/3+b^4/3)h^2\text{л}_n^2 + (23/60+$$
$$+3b^2/10 + 2b^4/15)h^4\text{л}_n^4),\ a_{23} = -\text{в}^6(2+(2+$$
$$+10b^2/3+4b^4/3)h^2\text{л}_n^2 + (7/12+19b^2/20+$$
$$+8b^4/15 + \text{в}^2b^2(1+2b^2+b^4/5)/3)h^4\text{л}_n^4)/2,$$

$$a_{43} = -\beta^4h^4b^2(1+2b^2+b^4/5)/24,\ b_0 =$$
$$=2a_{43}/a_0,\ a_{33} = -\text{в}^4(h^2(b^2(1+b^2/3)+(8b^4/$$
$$/5+3b^2+4/5)h^2\text{л}_n^2/12) + (1+2b^2+b^4/5)\text{в}^2\times$$
$$\times h^2\text{л}_n^2b^2/3),\ \|B_j, D_j, F_j\|(a,b,c) =$$
$$= \|B,D,F\|(b_{2j}, b_{1j}, b_{0j}), B = (a_1^2a + c - a_1b)(c_1 -$$
$$-b_1)/\text{Д}_0,\ D = (b_1^2a + c - b_1b)(a_1 - c_1)/\text{Д}_0,$$

$$F = (c_1^2a + c - c_1b)(b_1 - a_1)/\Delta_0,\ \Delta_0 = a_1^2(c_1 -$$
$$-b_1) + b_1^2(a_1-c_1) + c_1^2(b_1-a_1),\ b_{0k} = a_{0k} -$$
$$-a_{3k}m_2/a_0,\ b_{1k} = a_{1k} - a_{3k}l_2/a_0, b_{2k} = a_{2k} -$$
$$-a_{3k}k_2/a_0,\ (k=1,\ 2),\ b_{03} = a_{03} - m_2(a_{33} -$$
$$-a_{43}k_2/a_0, b_{13} = a_{13} - a_{43}m_2/a_0 - l_2(a_{33} - a_{43}\times$$
$$\times k_2/a_0)/a_0,\ b_{23} = a_{23} - a_{43}l_2/a_0 - k_2(a_{33} -$$
$$-a_{43}/a_0)/a_0,\ k_2 = (a_1+b_1+c_1)a_0,\ l_2 = (a_1b_1 +$$
$$+a_1c_1 + b_1c_1)a_0,\ m_2 = a_1b_1c_1a_0,\ s_1 = 0,$$

$$s_2 = i\alpha\sqrt{6/h^2+\lambda_n^2},\ s_3 = i\beta\sqrt{6/h^2+\lambda_n^2},$$

$$s_4 = i\sqrt{3\beta/h^2 + \alpha^2\lambda_n^2}.$$

The exponentials in (8) were laid out in a power series, which were deducted six first members. Here $J_0(t)$, $J_1(t)$ – Bessel functions of the first kind of zero and first order, respectively, and the function $\bar{J}_0(t)$ is defined as follows:

$$\bar{J}_0(t) = \int_0^t J_0(\tau)d\tau.$$

It is easy to verify that when the thickness of the layer tends to infinity $\lim_{h\to\infty}\phi_i(h,n,t)=0,\ \ (i=1,\ 2,\ 3)$ functions ϕ_i are zero and equation (13) coincides with the corresponding equity for the half-space [18, 20, 24].

Using last relation, the mixed boundary conditions (4) and (5) were satisfied. Forming a series expansion in the Eigen functions and equating coefficients of lie terms in $\cos\lambda_n x$, an infinite system of Volterra integral equations of the second kind was obtained:

$$V_n(t)+\sum_{m=0}^{\infty}\alpha_{mn}(x^*)\int_0^t V_m(\tau)F_m(t-\tau)d\tau =$$
$$=C_n(x^*)v_T(t), \tag{14}$$

where

$$\alpha_{mn}(x^*)=\frac{1}{N_n^2}\int_{x^*}^{l}\cos\lambda_m x\cos\lambda_n x dx,$$

$$C_n(x^*)=\frac{1}{N_n^2}\int_0^{x^*}\cos\lambda_n x dx,\ N_n^2=\int_0^l \cos^2\lambda_n x dx.$$

Transforming (7) and making use of (3), (10) and (11), we rewrite the equations of motion of the projectile in the form

$$\frac{dv_T(t)}{dt}=-\frac{2\alpha}{M}\Bigg\{v_T(t)x^*(t)+$$
$$+\sum_{n=0}^{\infty}\frac{\sin\lambda_n x^*(t)}{\lambda_n}\int_0^t V_n(\tau)F_n(t-\tau)d\tau\Bigg\}. \tag{15}$$

NUMERICAL IMPLEMENTATION

The numerical implementation of the governing system of equations (14), (15) is based on a combination of the quadrature and reduction methods. The integrals in Eqs. (13) − (15) are evaluated according to the Gregory symmetrical quadrature formula for equidistant fifth-order nodes [25]. The Cauchy problem for the differential equation (15) is solved by the fourth-order Adams method with local truncation error $O(\Delta t^6)$ [25], where Δt is the length of the subintervals into which the interval [0, T] is partitioned. The initial phase of the solution is computed in steps of $\Delta t/16$. The order of reduction N is chosen from considerations of practical convergence. To smooth the oscillations encountered in the summation of finite number of terms of the series and to offset the Gibbs phenomenon, an averaging operation [22] was introduced, which in the case of the sum of a finite number of terms of a trigonometric series reduces to term-by-term multiplication of terms of the finite sum by the Lanczos multipliers $\sigma_n=(\sin(n\pi/N))/(n\pi/N),\ (n=\overline{0,N})$.

Setting the shear modulus μ equal to zero, we obtain as a special case the problem of the impact of a shell on the surface of a fluid. An aluminium layer was considered as an example $V_0=0.003$, $\mu=0.3582K$, $h/R=0.02$, $M=0.25$, $h=2$, $T=2$, $d=0.05$ (Fig.1). Figures 2 – 5 shows the results of calculations: normal stresses σ_{zz} (Fig.2) and the normal displacements u_z (Fig.3) at the middle point of initial contact area, the reaction of the layer P (Fig. 5), the penetration rate V of the projectile into the medium of layer (Fig.4).

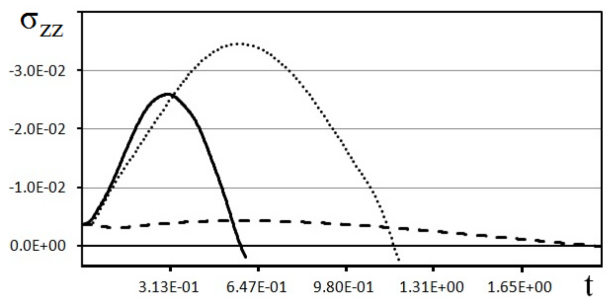

Fig. 2. The dependence of the normal stresses

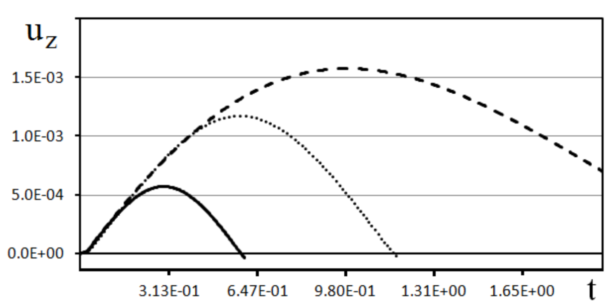

Fig. 3. The dependence of the normal displacement

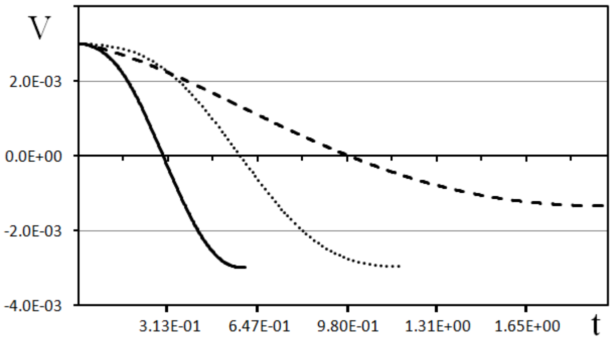

Fig. 4. The dependence of the velocity

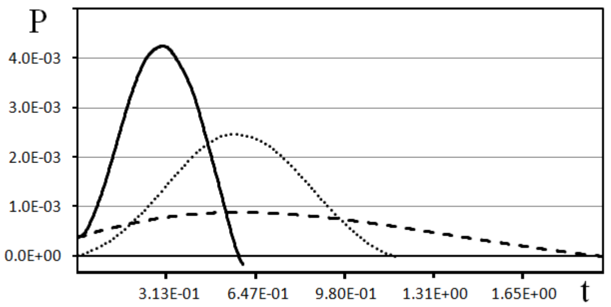

Fig. 5. The dependence of the elastic layer reaction

Solid curve corresponds to the case of the impact of hard cylinder with flat surface with width $2d$ on the elastic layer (Case 1). For comparison shown dash and dots curves which represent the cases of the impact of hard cylinder with flat surface $d = 0.05$ on the elastic half-space (Case 2) and hard cylinder $d = 0$ on the elastic layer (Case 3) corresponding. The maximal normal stresses σ_{zz} arise in case of the impact of hard cylinder on the elastic half-space. In Case 2 the normal stresses are significantly less than in Cases 1 and 3 and in Case 3 on 30% higher than in Case 1.

SUMMURY

Developed solution gives ability adequately simulate an impact processes of the impact of cylindrical flat body on the elastic layer especially when initial contact area is strip.

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Удар жесткого цилиндра с плоским срезом по упругому слою

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Аннотация. Исследуется напряженно-деформированное состояние упругого слоя, который деформируется в результате удара жесткого цилиндра с плоской площадкой. Начальный контакт происходит по поверхности плоского среза. Рассматривается краевая задача с динамически изменяющейся зоной контакта. Используется подход, который сводит уравнения динамики системы штамп-слой к бесконечной системе интегральных уравнений Вольтерра второго рода. Это дает возможность эффективно провести численный анализ задачи и достоверно определить динамические количественные и кинематические характеристики, описывающие процесс удара в зависимости от начальной скорости ударника и упругих параметров слоя.

Ключевые слова: удар, упругий, слой, плоская задача, жесткий цилиндр.