MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
Kyiv national university of construction and architecture

ANALYSIS OF PLANAR STATICALLY DETERMINATE BAR SYSTEMS

Methodical instructions
for performing of the calculation-graphic work
for students of the specialty 073 «Management»
of the education program «Management of organizations and administration»

Kyiv 2023

УДК 624.01/07
A11

Authors: I.D. Kara, PhD, Associate Professor

H.A. Zatyliuk, PhD

R.L. Stryhun, PhD

Reviewer O.V. Kostina, PhD, Associate Professor

Responsible for the edition V.V. Lizunov, DSc, Professor

Was approved on the meeting of the department of Structural Mechanics, the protocol №7 from 1 March 2023.

In author version.

A11 **Analysis** of planar statically determinate bar systems: methodical instructions for performing of the calculation-graphic work / I.D. Kara, H.A. Zatyliuk, R.L. Stryhun. – Kyiv: KNUCA, 2023. – 32 p.

Містять короткі теоретичні відомості, методичні вказівки, приклад індивідуального завдання та розрахунку статично визначуваних стержневих систем.

Призначено для студентів спеціальності 073 «Менеджмент» освітньої програми «Менеджмент організацій і адміністрування».

Includes short theoretical data, methodical instructions, the example of the individual task and the analysis of statically determinate bar systems: the truss, the beam and simple frames.

For students of the specialty 073 «Management» of the education program «Management of organizations and administration».

CONTENTS

Basic Principles

To solve comlex specialized tasks and practical problems that arise in a process of construction organizations functioning leaders and managers must have knowledge of a field of construction production and methods of engineering structures analysis. Structural mechanics is the basic discipline in the field of structures designing. It develops methods of analysis of strength, stability and stiffness of engineering constructions and structures. Knowledge, that were received when studying this discipline, future managers can use in further taking of courses of Building Structures and in a future practical activity.

Methodical instructions were designed to help seekers of the education program «Management of organizations and administration» to perform the calculation-graphic work. Necessary theoretical material is presented in educational literature also [1-5]. Methodical instructions consist of: short theoretical data, the example of the individual task and the example of the analysis of statically determinate bar systems.

The calculation-graphic work is made out by the seeker tidily on the one side of A4 format papers. The title page is the first page of the work. After the title page the given task is inserting, after which the work is stapled.

Short Theoretical Data

A real building is brought to simplified view by ignoring less important factors during the analysis of strength, stiffness and stability. This scheme is **an analytical model** of a structure.

External loads and influences act on a engineering structure during operation that causes deformation of its elements and occurrence of displacements.

A force is a physical quantity that defines a quantitative mearure of a bodies mechanical interaction between each other. In the International System of Units (SI) the unit of a force is newton – $1N$ ($1kN=10^3N$). A totality of forces that act on a body is a system of forces (a force system) (Fig. 1).

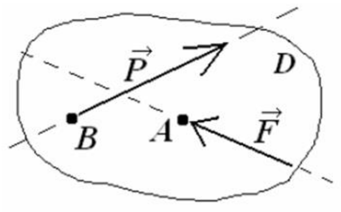


Fig. 1

Whereas a force is a vector quantity then it's defined on a plane by: the modulus (the absolute value), the direction and the line of action. To determine a force analytically is necessary first to choose the coordinate system in which force projections will be determined. Usually the Cartesian coordinate system with mutually perpendicular axes x and y in the plane is used. A force projection on an axis is a product of the force modulus by the cosine of the guiding angle to the corresponding axis (Fig. 2).

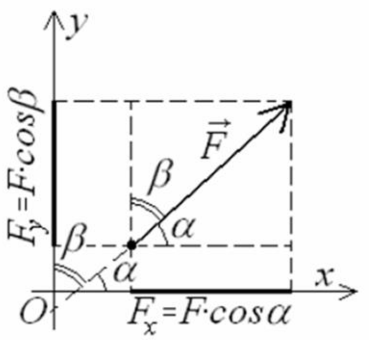


Fig. 2

A uniformly distributed load – a continuous load of a constant intensity that is applied at a certain length of a structure (Fig. 3). This type of load has the unit of the force unit related to the length unit: N/m (kN/m).

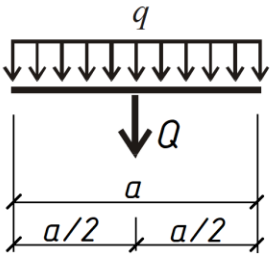


Fig. 3

The resultant to a uniformly distributed load Q (Fig. 3) is equal to the product of the load intensity q by the length a along which it's distributed:

$$Q = q \cdot a . \tag{1}$$

A moment about a dot is a product of the force modulus by the arm. **An arm** is the the smallest distance from the point about which the moment is determined to the the force line of action.

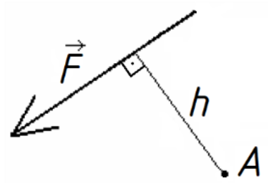


Fig. 4

The moment of the force $\vec{F}$ about the point A is:

$$M_A\left(\vec{F}\right) = \pm F \cdot h, \tag{2}$$

where h is the arm from the point A to the line of action of the force $\vec{F}$ (Fig. 4).

When writing an equation of the algebraic sum of moments about a dot (a centre of a moment) a clockwise moment is accepted positive and a counterclockwise moment is accepted negative; or vise versa.

A moment of the force has the unit of the product of the force unit by the length unit – $N{\cdot}m$ ($kN{\cdot}m$). If the force $\vec{F}$ is moved along its line of action then its moment about the point A will remain unchanged. If the line of action of the force $\vec{F}$ passes through the point A then the moment of the force about this point will be equal to zero.

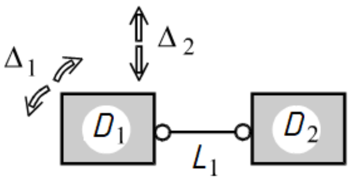
A static analysis of a structure consists in determination of forces which arise in its elements under the action of external loads which are applied to this structure. It can be divided into two stages:

1) determination of reaction forces in restraints including support reactions (the external problem);

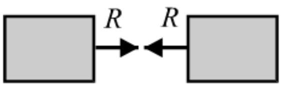
2) calculation of internal forces: bending moments, shear and axial forces which arise in elements of a structure during the deformation process (the internal problem).

Structural elements of analytical models are rigid bodies and restraints. **A rigid body** (a disc) is the part of the analytical model that is predetermined or previously demonstrated geometry stable. An entire system also can be the rigid body if it definitely is geometry stable. **A connecting restraint** (a connection) is the element of an analytical model that restricts in a certain way the mutual movements of rigid bodies, which are connected by this device, reducing the degrees of freedom of a system. There are such kinds of restraints in planar analytical models: a link, a simple hinge, a coupling, a simple rigid joint. If a restraint connects the structure or its member to the foundation than it is defined as **a support restraint** (a support).

A link L_1 (Fig. 5*a*) is the connecting restraint that connects two rigid bodies and restricts the linear movement of the one rigid body relative to the other along the axial of a link. Such connection allows the mutual rotation Δ_1 of rigid bodies and the linear movement Δ_2 along the normal of a link reducing one degree of freedom. The reaction force R arises in a link the line of action of which passes through points of the hinged connection of a link and rigid bodies (Fig. 5*b*).



a



b

Fig. 5

The support restraint that is corresponding to a link is **a roller support** (Fig. 6*a*,*b*,*c*) in which the reaction force R_A arises (Fig. 6*d*).

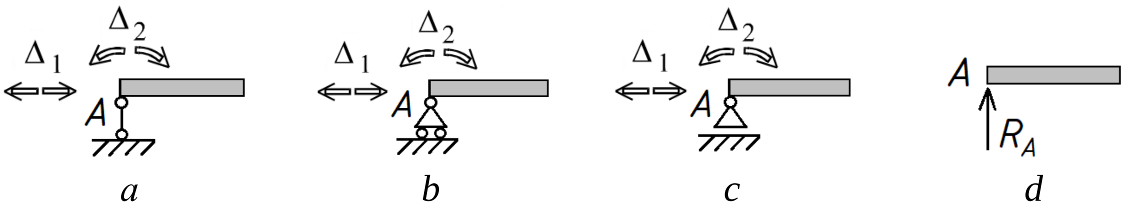
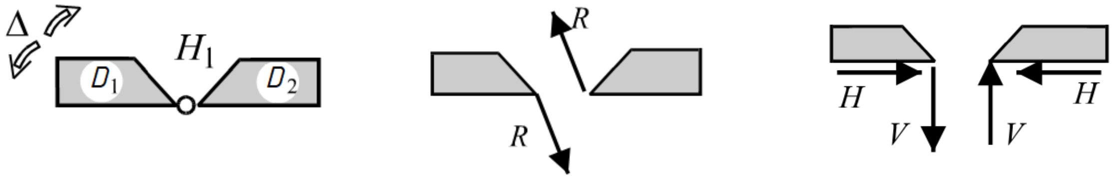


Fig. 6

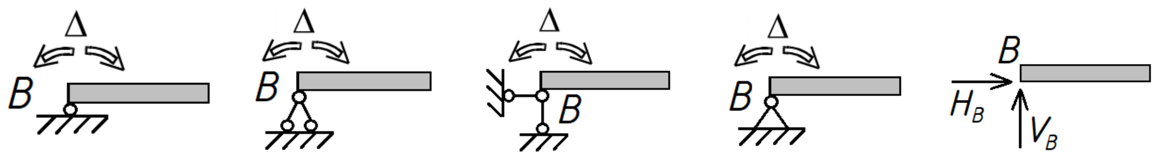
A simple hinge H_1 (Fig. 7*a*) is the connection of two rigid bodies that restricts mutual linear movements of these rigid bodies allowing their mutual rotation Δ about the axis that is moved through the hinge centre. This connection reduces two degrees of freedom. The reaction force R arises in a simple hinge the line of action of which passes through the device centre but has the previously unknown direction (Fig. 7*b*). That's why when performing analyses it is usually replaced by two components: horizontal H and vertical V (Fig. 7*c*).



a *b* *c*

Fig. 7

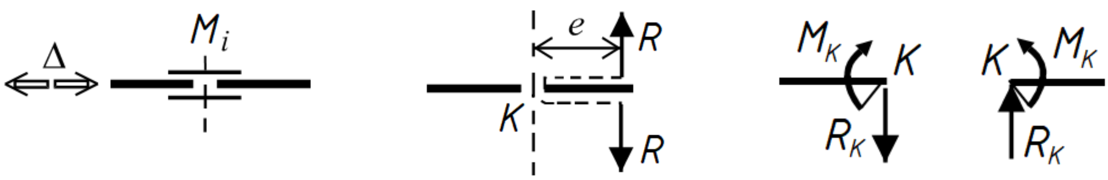
Equivalent to a simple hinge is **a hinged support** (Fig. 8*a*,*b*,*c*,*d*). The reaction force also occurs in it that can be replaced by horizontal and vertical components: H_B and V_B (Fig. 8*e*).



a *b* *c* *d* *e*

Fig. 8

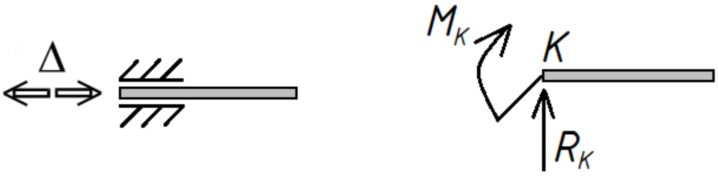
A coupling M_1 (Fig. 9*a*) is the device that connects two rigid bodies and allows the linear movement Δ along the fixed straight line. This connection reduces two degrees of freedom. The reaction force R arises in a coupling the line of action of which is perpendicular to the possile movement and passes with the eccentricity e to the coupling centre K (Fig. 9*b*). In analytical calculations this reaction is presented as two components: the reaction force R_K and the reaction moment (the couple of moment) M_K which act in the connection centre (Fig. 9*c*).



a *b* *c*

Fig. 9

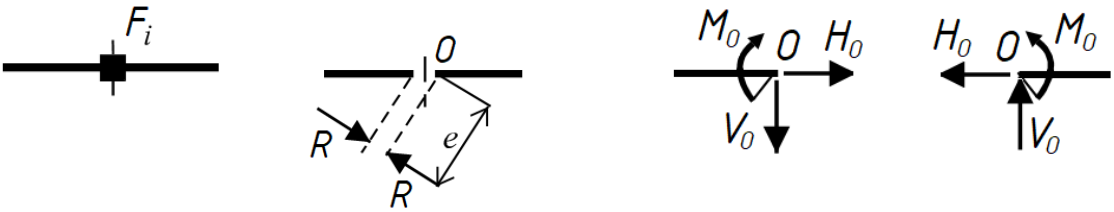
A directional support (Fig. 10*a*) is the support that is equivalent to a coupling. The support reaction in the directional support is similarly presented as the force and the reaction moment (Fig. 10*b*).



a *b*

Fig. 10

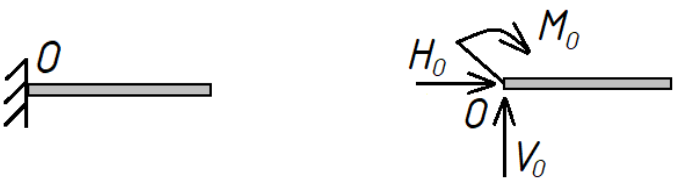
A simple rigid joint F_i (Fig. 11*a*) is the connection of two rigid bodies that restricts their mutual linear and rotation movements. This device reduces three degrees of freedom. The line of action and the eccentricity *e* to the rigid joint centre *O* (Fig. 11*b*) of the reaction *R* in a simple rigid joint is previously unknown. That's why conveniently to represent it as three components which act in the centre of the connection device: the horizontal reaction H_O, the vertical reaction V_O and the reaction moment (the couple of moment) M_O (Fig. 11c).



a *b* *c*

Fig. 11

The support device that is identical to a simple rigid joint is **a fixed support** (Fig. 12*a*) the reaction in which also conveniently to replace by two mutually perpendicular reactions and the reaction moment (Fig. 12*b*).



a *b*

Fig. 12

To determine support reaction and restraint forces can write three independent equilibrium equations. If a planar force system is in equilibrium than the algebraic sum of projections of all forces on two mutually perpendicular axes and the algebraic sum of moments of all forces about any point on the plane must be equal to zero:

$$\begin{cases} \sum_{i=1}^{n} F_{ix} = 0, \\ \sum_{i=1}^{n} F_{iy} = 0, \\ \sum_{i=1}^{n} M_K\left(F_i\right) = 0. \end{cases} \tag{3}$$

When solving some problems is conveniently to replace one or two equations of projections of forces on axes by equations of moments about a point:

$$\begin{cases} \sum_{i=1}^{n} F_{ix} = 0, \\ \sum_{i=1}^{n} M_B\left(F_i\right) = 0, \\ \sum_{i=1}^{n} M_K\left(F_i\right) = 0. \end{cases} \quad \text{or} \quad \begin{cases} \sum_{i=1}^{n} M_A\left(F_i\right) = 0, \\ \sum_{i=1}^{n} M_B\left(F_i\right) = 0, \\ \sum_{i=1}^{n} M_K\left(F_i\right) = 0. \end{cases}$$

In the last system all thee points A, B and K should not lie on one straight line.

Structural mechanics, in the narrow sense, considers **bar systems** – mechanical systems which consist of bars. Such systems are, in particular: trusses, beams and frames.

A truss is a bar system which is composed of straight bars connected between each other at their ends by hinged joints (flexible connections) (Fig. 13).

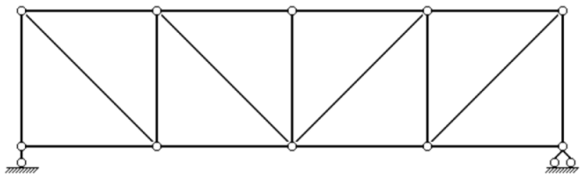


Fig. 13

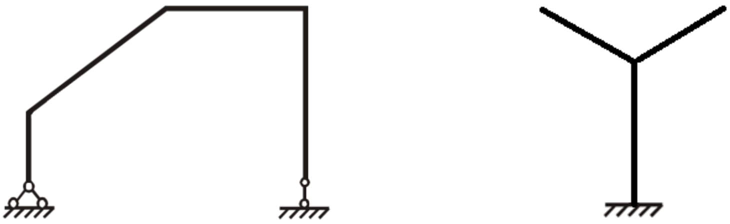
Loads on trusses are applied only in joints as a result of which only axial forces arise in truss members. There are two the most common methods for determination of these forces: the method of joints, considering joints as free bodies, and the method of sections, taking a desired portion of the truss as a free body.

A beam is a bar structure the length of which is much larger than the width and the depth and which works mostly on bending (Fig. 14*a*,*b*).

a *б*

Fig. 14

A structure which consists of straight bars connected between each other rigidly at all or some joints is **a frame** (rigid frames) (Fig. 15*a*,*b*).



a *б*

Fig. 15

Such internal forces exist at sections of beams and frames: the bending moment *M*, the shear force *Q* and the axial force *N*. Customarily the beam is under the action of vertical loads that's why axial forces at its sections are equal to zero. The diagram of internal forces distribution along the length of the bar is termed **an internal force diagram**.

The bending moment *M* at the section *k* is equal in magnitude to the algebraic sum of moments about the centroid of the cross section of all external loads acting on one side of the section under consideration:

$$M_k = \sum M_k^{left} \text{ or } M_k = \sum M_k^{right}.$$

The shear force *Q* at the section *k* is equal to the algebraic sum of projections of all external loads in the direction of the normal *n* to the axis of the bar under consideration on one side of the section:

$$Q_k = \sum F_{nk}^{left} \text{ or } Q_k = \sum F_{nk}^{right}.$$

The axial force *N* at the section *k* is equal to the algebraic sum of projections of all external loads in the direction of the tangent *s* to the axis of the bar on one side of the section under consideration:

$$N_k = \sum F_{sk}^{left} \text{ or } N_k = \sum F_{sk}^{right}.$$

Magnitudes of internal forces on diagrams should be depicted perpendicular to the axial line of the member which the section belongs to. Ordinates of a bending moments diagram are depicted on the tension side of the member. The shear force is positive if makes the member rotate clockwise. If the summary axial force makes the member in tensile state than it has the positive sign. Ordinates of shear and axial forces diagrams can be depicted on any sides of the bar with sign indication in a circle on every member. Hatching of internal forces diagrams are done with thin lines perpendicular to the axial line of members.

After construction it is need to check the correctness of internal forces diagrams. **The check of the equilibrium of free bodies of frame's joints** consists in the check of employing of equilibrium conditions for system parts isolated by cross-sections that are located at the infinitesimally small distance from members connection points (members ends). **The check of the relationship between *M* and *Q* diagrams** is performed for beams and frames. It consists in determination of shear forces values using ordinates of a bending moments diagram: the magnitude of Q force is equal to the tangent of the slope angle of the diagram M to the axis of the member. The shear force is positive if the diagram M deflects from the member's axis clockwise on segments where the bending moments diagram has a straight line character. To verify the bending moments diagram on segments, where it is curvilinear, the isolated member of the system with applied loads, moments of member's ends and unknown shear forces is necessary to consider. If the diagram Q changes the sign, at this section the bending moments diagram should has the extreme.

To determinate a displacement the virtual system is necessary to create removing all external loads and applying the unit load that corresponds to the desired displacement: the unit horizontal load corresponds to the horizontal displacement; the unit vertical load corresponds to the vertical displacement. Deformations of frames are mainly caused by bending moments therefore is enough to depict only the bending moments diagram M_1 and to calculate the displacement Δ_{1P} using the simplified Maxwell-Mohr's formula:

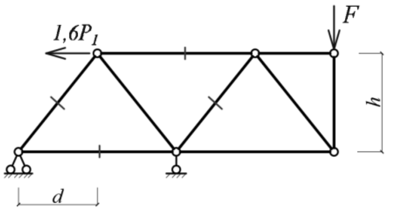
$$\Delta_{1P} = \sum \int_l \frac{M_1 \cdot M}{EI} dx, \qquad (4)$$

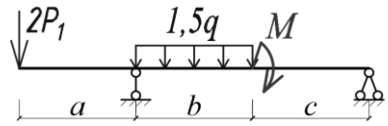
where l is the length of the integration segment.

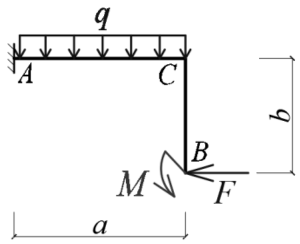
The Example of the Individual Task

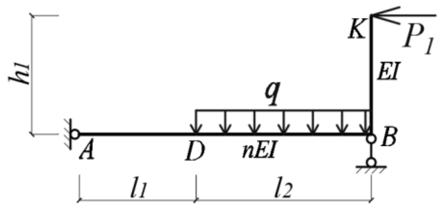
(the front side of the task)

1









Evaluate the horizontal displacement of the point K

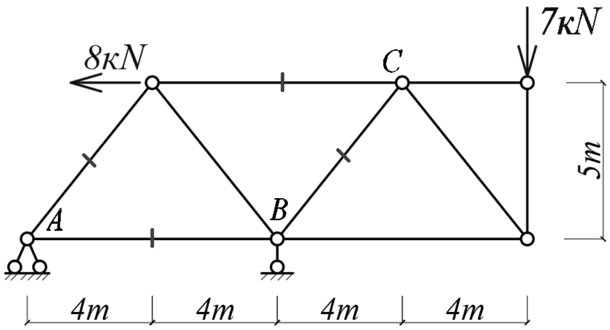
d, m	*h, m*	*a, m*	*b, m*	*c, m*	*l_1, m*	*l_2, m*	*h_1, m*	*F, kN*	*M, kNm*	*q, kN/m*	*P_1, kN*	*n*
2	2	4	2	6	2	5	3	5	4	3	8	**2**
4	**5**	5	3	**5**	**4**	**6**	**4**	3	6	6	10	3
2	4	**6**	**4**	4	6	4	7	9	5	8	12	4
5	5	2	5	3	3	2	4	**7**	**9**	4	14	2
2	3	3	7	4	2	1	5	1	7	**2**	**5**	3

Underlined values in the table mean data that are defined to every student by the university teacher for further calculations.

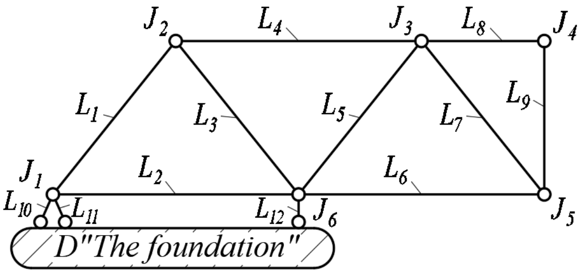
(the back side of the task)

Contents of the calculation-graphic work
The Analysis of the Truss 1. To perform the kinematic analysis. 2. To find support reactions and to determine internal forces in marked members. **The Analysis of the Simply Supported Beam** 1. To perform the kinematic analysis. 2. To find support reactions. 3. To construct internal forces diagrams of bending moments and shear forces. 4. To perform the check of the relationship between diagrams *M* and *Q*. **The Analysis of Frames** 1. To perform the kinematic analysis. 2. To find support reactions. 3. To construct bending moments, shear and axial forces diagrams. 4. To perform checks of diagrams. 4. For simply supported frame: — to create the virtual system that corresponds to the desired displacement; — to construct the bending moments diagram of virtual system; — to calculate the preset displacement using Maxwell-Mohr's formula.

The Example of the Performing of the Individual Task
The Analysis of the Truss



1. <u>The kinematic analysis.</u>



1.1 The quantitative stage:

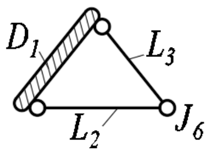
$J = 0$, $L = 3$.

$G = 2 \cdot J - L = 2 \cdot 6 - 12 = 0$.

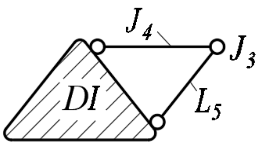
The system has the minimally required quantity of restraints.

1.2 The quantitative stage:

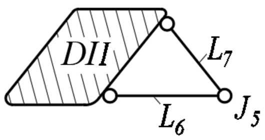
Represent the bar L_1 as the rigid body D_1:



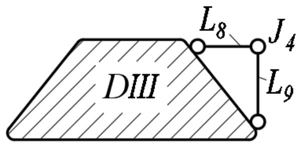
$$\frac{D_1 + J_6}{L_2,\, L_3} = D\,I$$ (the connection «diad»);



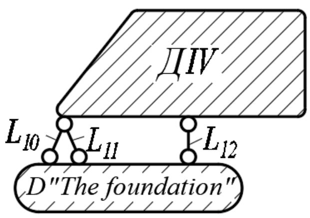
$$\frac{D\,I + J_3}{L_4,\, L_5} = D\,II$$ (the connection «diad»);



$$\frac{D\,II + J_5}{L_6,\, L_7} = D\,III$$ (the connection «diad»);



$$\frac{D\,III + J_4}{L_8,\, L_9} = D\,III$$ (the connection «diad»);

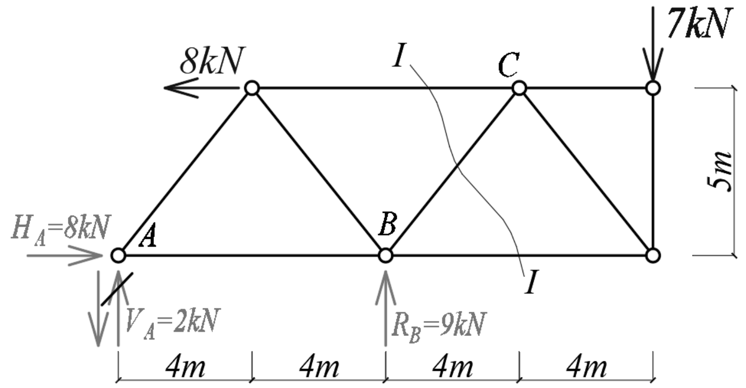


$$\frac{D\,IV + D\text{"}the\ foundation\text{"}}{L_{10},\, L_{11},\, L_{12}} = D\,IV$$

(the Shuhov’s connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>



$\sum F_x = 0:\ H_A - 8 = 0;\ H_A = 8kN$.

$\sum M_A = 0:\ -8\cdot 5 + 7\cdot 16 - R_B \cdot 8 = 0;\quad R_B = \dfrac{72}{8} = 9kN$.

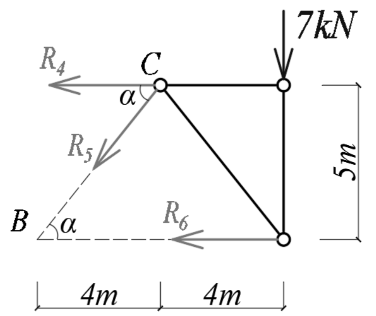
$\sum F_y = 0:\ V_A - 7 + 9 = 0;\quad V_A = -2kN$.

The check:

$\sum M_C = -8\cdot 5 - 2\cdot 12 + 9\cdot 4 + 7\cdot 4 = 64 - 64 = 0.$

3. <u>Determination of internal forces in marked members.</u>

Perform the cross section *I-I* and consider the right portion:



$\sum M_B = 0:\ -R_4 \cdot 5 + 7 \cdot 8 = 0;$

$R_4 = \dfrac{56}{5} = 11,2kN$.

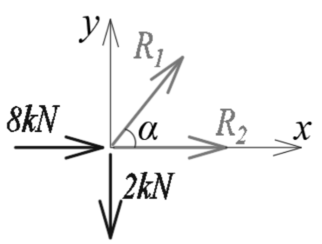
$\alpha = arctg\,\dfrac{5}{4} = 51,34^o$

$\cos\alpha = 0,6247;\ \sin\alpha = 0,7809.$

$\sum F_y = 0:\ -R_5 \cdot \sin\alpha - 7 = 0;$

$R_5 = -\dfrac{7}{0,7809} = -8,964kN$.

Considering the joint *A* as free body:



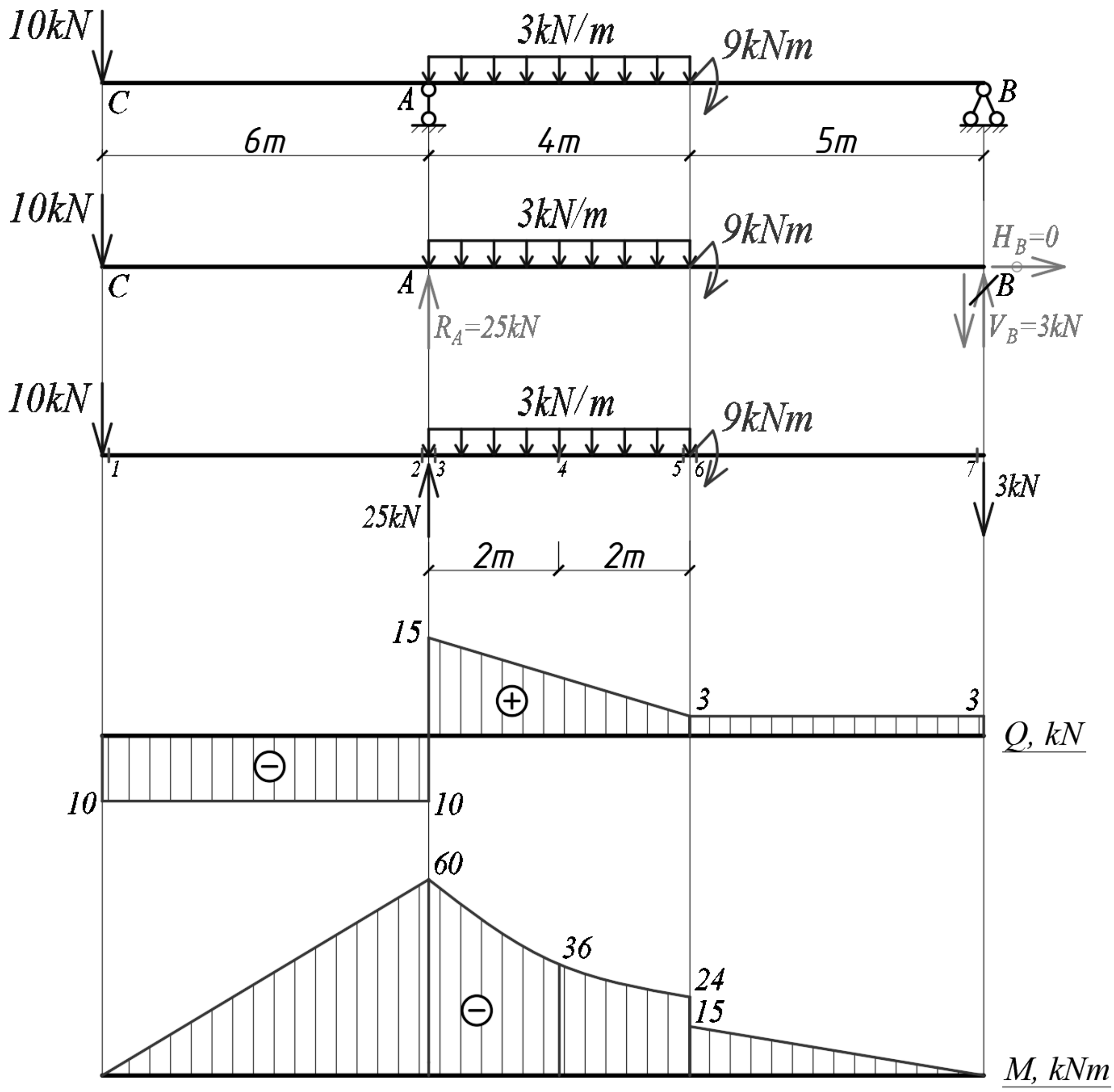
$\sum F_y = 0:\ -2 + R_1 \cdot \sin\alpha = 0;$

$R_1 = \dfrac{2}{0,7809} = 2,561kN$.

$\sum F_x = 0:\ 8 + R_2 + 2,561 \cdot \cos\alpha = 0;$

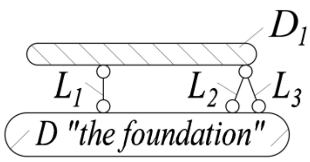
$R_2 = -9,6kN$.

The Analysis of the Simply Supported Beam



1. <u>The kinematic analysis.</u>

1.1 The quantitative stage:



$D = 2,\ J = 0,\ F = 0,\ H = 0,\ L = 3.$

$G = 3 \cdot D + 2 \cdot J - 3 \cdot F - 2 \cdot H - L - 3 =$

$= 3 \cdot 2 + 2 \cdot 0 - 3 \cdot 0 - 2 \cdot 0 - 3 - 3 = 6 - 6 = 0.$

The system has the minimally required quantity of restraints.

1.2 The quantitative stage:

$$\frac{D_1 + D\text{"the foundation"}}{L_1,\ L_2,\ L_3} = DI$$ (the Shuhov's connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>

$\sum F_x = 0:\ H_B = 0$.

$\sum M_A = 0:\ -10\cdot 6 + (3\cdot 4)\cdot 2 + 9 - V_B\cdot 9 = 0;\quad R_B = -\dfrac{27}{9} = -3kN$.

$\sum F_y = 0:\ -10 + R_A - (3\cdot 4) - 3 = 0;\quad R_A = 25kN$.

The check:

$\sum M_C = -25\cdot 6 + (3\cdot 4)\cdot 8 + 9 + 3\cdot 15 = 150 - 150 = 0.$

3. <u>Depiction of internal forces diagrams.</u>

$Q_1 = -10kN,$

$Q_2 = -10kN,$

$Q_3 = -10 + 25 = +15kN,$

$Q_5 = -10 + 25 - (3\cdot 4) = +3kN,$

$Q_6 = +3kN,$

$Q_7 = +3kN.$

$M_1 = 0,$

$M_2^{left} = -10\cdot 6 = -60kNm,$

$M_3^{left} = -10\cdot 6 = -60kNm,$

$M_4^{left} = -10\cdot 8 + 25\cdot 2 - (3\cdot 2)\cdot 1 =$
$= -36kNm,$

$M_5^{left} = -10\cdot 10 + 25\cdot 4 - (3\cdot 4)\cdot 2 =$
$= -24kNm,$

$M_6^{left} = -24 + 9 = -15kNm,$

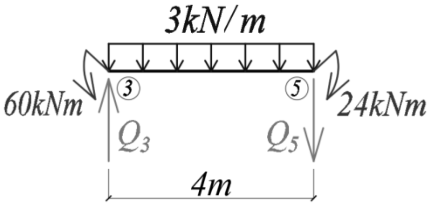
$M_7 = 0.$

4. <u>The check of the relationship between diagrams *M* and *Q*.</u>

$Q_{1-2} = -\dfrac{60-0}{6} = -10kN;$

$Q_{6-7} = +\dfrac{15-0}{5} = +3kN;$

Q_{3-5}:



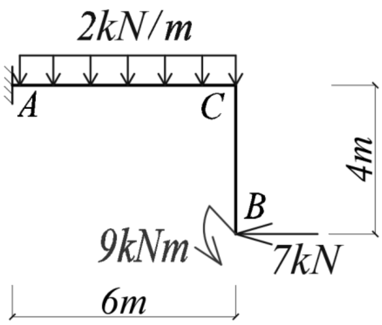
$\sum M_3 = 0:\ -60 + (3\cdot 4)\cdot 2 + Q_5\cdot 4 + 24 = 0;$

$Q_5 = +\dfrac{12}{4} = +3kN.$

$\sum M_5 = 0:\ -60 + Q_3\cdot 4 - (3\cdot 4)\cdot 2 + 24 = 0;$

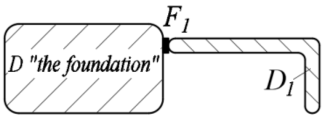
$Q_3 = +\dfrac{60}{4} = +15kN.$

The Analysis of Overhanging Frame



1. <u>The kinematic analysis.</u>

1.1 The quantitative stage:



$D = 2,\ J = 0,\ F = 1,\ H = 0,\ L = 0.$

$G = 3\cdot D + 2\cdot J - 3\cdot F - 2\cdot H - L - 3 =$

$= 3\cdot 2 + 2\cdot 0 - 3\cdot 1 - 2\cdot 0 - 0 - 3 = 6 - 6 = 0.$

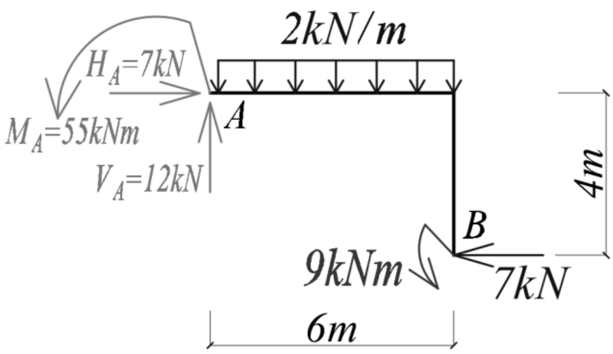
The system has the minimally required quantity of restraints.

1.2 The quantitative stage:

$$\frac{D_1 + D\text{"}the\ \ foundation\text{"}}{F_1} = DI$$ (the fixed support connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>



$\sum F_x = 0:\ H_A - 7 = 0;\ H_A = 7kN.$

$\sum F_y = 0:\ V_A - (2\cdot 6) = 0;$

$V_A = 12kN.$

$\sum M_A = 0:$

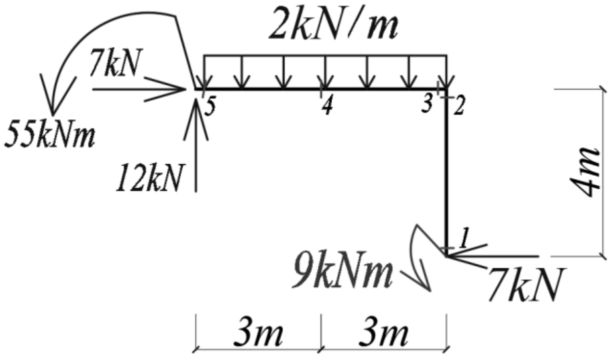
$-M_A + (2\cdot 6)\cdot 3 + 7\cdot 4 - 9 = 0;$

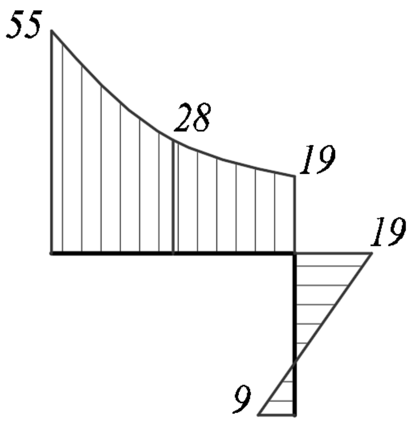
$M_A = 55kNm.$

The check:

$\sum M_B = -55 + 7\cdot 4 + 12\cdot 6 - (2\cdot 6)\cdot 3 - 9 = 100 - 100 = 0.$

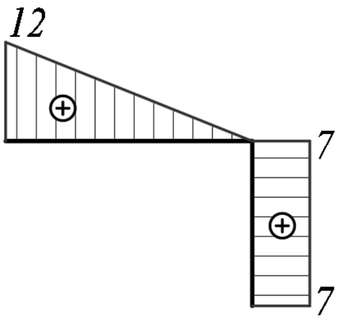
3. <u>Depiction of internal forces diagrams.</u>





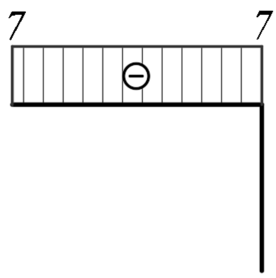
<u>M, kNm</u>

$$M_1 = -9kNm,$$
$$M_2^{low} = -9 + 7 \cdot 4 = +19kNm,$$
$$M_3^{right} = -9 + 7 \cdot 4 = +19kNm,$$
$$M_4^{right} = -9 + 7 \cdot 4 + (2 \cdot 3) \cdot 1{,}5 = \\ = +28kNm,$$
$$M_5^{right} = -9 + 7 \cdot 4 + (2 \cdot 6) \cdot 3 = \\ = +55kNm.$$



<u>Q, kN</u>

$$Q_1 = +7kN,$$
$$Q_2 = +7kN,$$
$$Q_3 = 0;$$
$$Q_5 = +(2 \cdot 6) = +12kN.$$

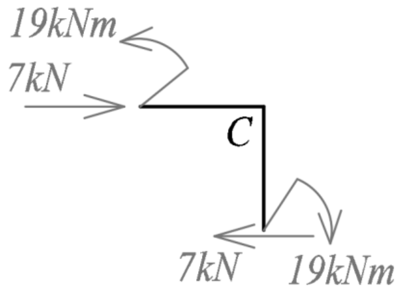


<u>N, kN</u>

$$N_1 = 0,$$
$$N_2 = 0,$$
$$N_3 = -7kN,$$
$$N_5 = -7kN.$$

4. <u>The check of the equilibrium of free bodies of frame's joints.</u>

The joint *C*:

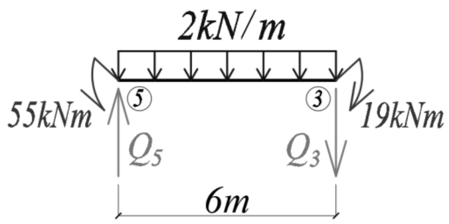


$$\sum F_x = 7 - 7 = 0,$$
$$\sum F_y = 0,$$
$$\sum M_C = -19 + 19 = 0.$$

5. <u>The check of the relationship between *M* and *Q* diagrams.</u>

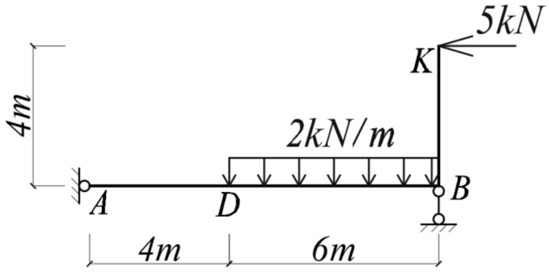
$$Q_{1-2} = +\frac{19-(-9)}{4} = +7kN;$$

Q_{3-5}:



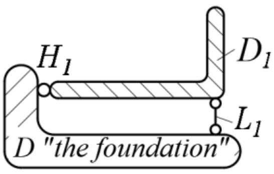
$$\sum M_5 = 0:$$
$$-55 + (2\cdot 6)\cdot 3 + Q_3 \cdot 6 + 19 = 0;$$
$$Q_3 = \frac{55-55}{6} = 0.$$
$$\sum M_3 = 0:$$
$$-55 + Q_5 \cdot 6 - (2\cdot 6)\cdot 3 + 19 = 0;$$
$$Q_2 = +\frac{72}{6} = +12kN.$$

The Analysis of Simply Supported Frame



1. <u>The kinematic analysis.</u>

1.1 The quantitative stage:



$Д = 2,\ В = 0,\ П = 0,\ Ш = 1,\ С = 1.$

$G = 3 \cdot D + 2 \cdot J - 3 \cdot F - 2 \cdot H - L - 3 =$

$= 3 \cdot 2 + 2 \cdot 0 - 3 \cdot 0 - 2 \cdot 1 - 1 - 3 = 6 - 6 = 0.$

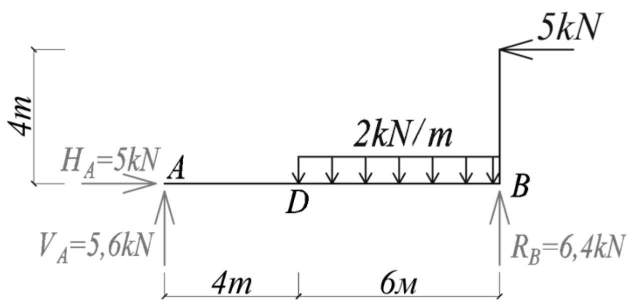
The system has the minimally required quantity of restraints.

1.2 The quantitative stage:

$\dfrac{Д_1 + Д\text{"земля"}}{Ш_1,\ С_1} = Д\, I$ (the Polonso's connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>



$\sum F_x = 0:\ H_A - 5 = 0;\ H_A = 5кН.$

$\sum M_A = 0:$

$(2 \cdot 6) \cdot 7 - 5 \cdot 4 - R_B \cdot 10 = 0;$

$$R_B = \frac{64}{10} = 6,4кН.$$

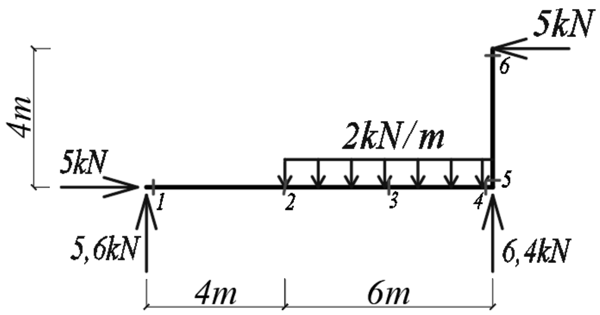
$\sum F_y = 0:\ V_A - (2 \cdot 6) + 6,4 = 0;$

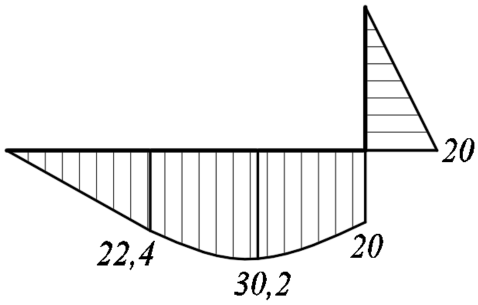
$$V_A = 5,6кН.$$

The check:

$\sum M_D = 5,6 \cdot 4 + (2 \cdot 6) \cdot 3 - 6,4 \cdot 6 - 5 \cdot 4 = 58,4 - 58,4 = 0.$

3. <u>Depiction of internal forces diagrams.</u>





M, kNm

$M_1 = 0,$

$M_2^{left} = +5,6 \cdot 4 = +22,4kNm,$

$M_3^{left} = +5,6 \cdot 7 - (2 \cdot 3) \cdot 1,5 =$

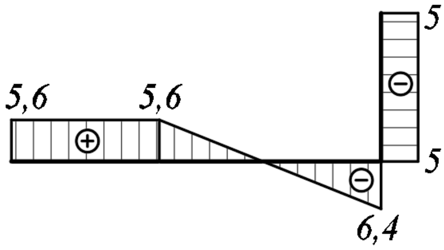
$= +30,2kNm,$

$M_4^{left} = +5,6 \cdot 10 - (2 \cdot 6) \cdot 3 =$

$= +20kNm,$

$M_6 = 0,$

$M_5^{up} = -5 \cdot 4 = -20kNm.$



Q, kN

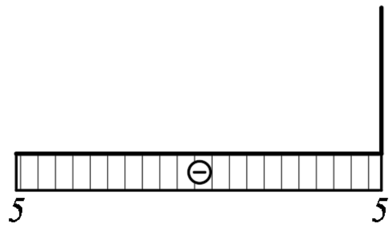
$Q_1 = +5,6kN,$

$Q_2 = +5,6kN,$

$Q_4 = +5,6 - (2 \cdot 6) = -6,4kN,$

$Q_6 = -5kN,$

$Q_5 = -5kN.$



N, kN

$N_1 = -5kN,$

$N_2 = -5kN,$

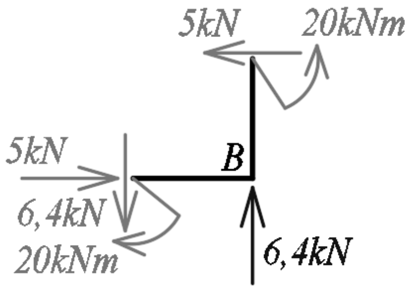
$N_4 = -5kN,$

$N_6 = 0,$

$N_5 = 0.$

4. The check of the equilibrium of free bodies of frame's joints.

The joint *B*:



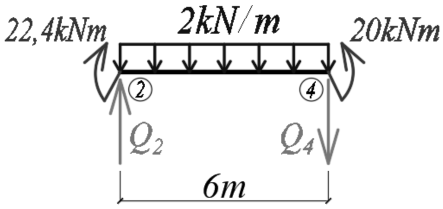
$$\sum F_x = 5 - 5 = 0,$$
$$\sum F_y = -6,4 + 6,4 = 0,$$
$$\sum M_B = 20 - 20 = 0.$$

5. The check of the relationship between *M* and *Q* diagrams.

$$Q_{1-2} = +\frac{22,4-0}{4} = +5,6kN;$$

$$Q_{5-6} = -\frac{20-0}{4} = -5kN;$$

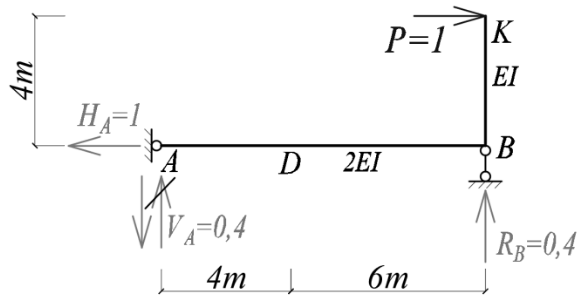
Q_{2-4}:



$$\sum M_2 = 0:$$
$$22,4 + (2\cdot 6)\cdot 3 + Q_4 \cdot 6 - 20 = 0;$$
$$Q_4 = -\frac{38,4}{6} = -6,4kN.$$
$$\sum M_4 = 0:$$
$$22,4 - (2\cdot 6)\cdot 3 + Q_2 \cdot 6 - 20 = 0;$$
$$Q_2 = +\frac{33,6}{6} = +5,6kN.$$

6. Evaluation of the preset displacement (the horizontal displacement of the point K).

Create the virtual system that corresponds to the desired displacement:



$$\sum F_x = 0:\ -H_A + 1 = 0;\ H_A = 1.$$

$$\sum M_A = 0:\ 1\cdot 4 - R_B \cdot 10 = 0;$$

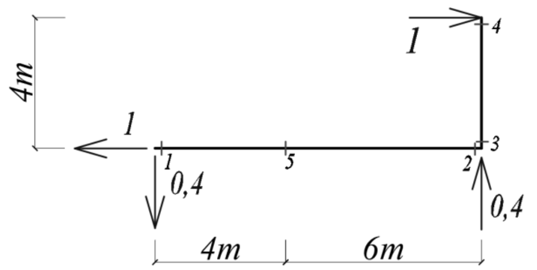
$$R_B = \frac{4}{10} = 0,4.$$

$$\sum F_y = 0:\ V_A + 0,4 = 0;\ V_A = -0,4.$$

The check:

$$\sum M_D = -0,4\cdot 4 - 0,4\cdot 6 + 1\cdot 4 = 4 - 4 = 0.$$

Depict the bending moments diagram of the virtual system:



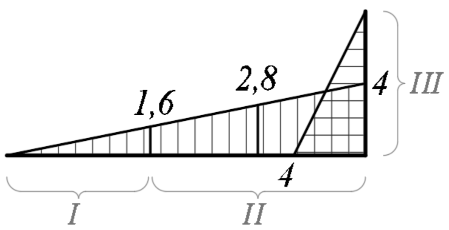
$$M_1 = 0,$$

$$M_2^{left} = -0,4\cdot 10 = -4m,$$

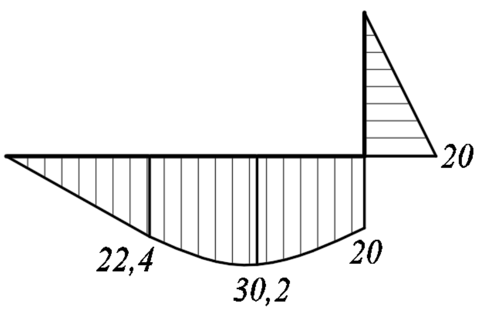
$$M_4 = 0,$$

$$M_3^{up} = +1\cdot 4 = +4m,$$

$$M_5^{left} = -0,4\cdot 4 = -1,6m.$$



M_1, m



M, kNm

$$\Delta_{1P} = \sum \int \frac{M_1 \cdot M}{EI} dx = -\frac{1}{2EI}\left(\frac{1}{2}\cdot 1,6\cdot 4\cdot \frac{2}{3}\cdot 22,4\right) +$$

$$+\frac{1}{2EI}\frac{6}{6}\left(-1,6\cdot 22,4 - 4\cdot 2,8\cdot 30,2 - 4\cdot 20\right) - \frac{1}{EI}\left(\frac{1}{2}\cdot 4\cdot 4\cdot \frac{2}{3}\cdot 20\right) =$$

$$+\frac{1}{EI}\left(-23,893 - 227,04 - 106,667\right) = \frac{1}{EI}\left(-357,6\right).$$

REFERENCES

1. *Bazhenov, V.A., Perelmuter, A.V., Shyshov, O.V.* (2013). Structural Mechanics. Computer Technology and Modeling: the textbook. Kyiv: VIPOL, 896 p. [in Ukrainian]

2. *Bazhenov, V.A., Ivanchenko, G.M., Shyshov, O.V., Pyskunov, S.O.* (2013). Structural Mechanics. Calculated Exercises. Problems. Computer Testing: the educational manual. Kyiv: Karavela, 439 p. [in Ukrainian]

3. *Vorona, Yu.V., Kara, I.D.* (2020). The Analysis of Statically Determinate Bar Systems: methodical instructions for performing of the calculation-graphic works. Kyiv: KNUCA, 28 p. [in Ukrainian]

4. *Kara, I.D., Kostina, O.V.* (2022). The Analysis of the Three Hinged Arch and Statically Determinate Frames: methodical instructions for performing of the calculation-graphic works. Kyiv: KNUCA, 32 p. [in Ukrainian]

5. *Shihua, B.,* Yaoqing, G. (2005). Structural Mechanics. Wuhan University of Technology Press, 627 p.

Educational and methodical edition

ANALYSIS OF PLANAR STATICALLY DETERMINATE BAR SYSTEMS

Methodical instructions
for performing of the calculation-graphic work
for students of the specialty 073 «Management»
of the education program «Management of organizations and administration»

Authors: **Kara** Iryna Dmytrivna
Zatyliuk Herman Anatoliiovych
Stryhun Ruslan Leonidovych

Комп’ютерне верстання *А.П. Селівестрової*

Підписано до друку 07.12. 2023. Формат 60 × 84 1/ 16.
Ум. друк. арк. 1,63. Обл.-вид. арк. 1,75.
Електронний документ. Вид. № 131/III-23

Видавець і виготовлювач
Київський національний університет будівництва і архітектури

Повітрофлотський проспект, 31, Київ, Україна, 03037

Свідоцтво про внесення до Державного реєстру суб’єктів
видавничої справи ДК № 808 від 13.02.2002