MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
Kyiv national university of construction and architecture

THE ANALYSIS OF THE THREE HINGED ARCH AND STATICALLY DETERMINATE FRAMES

Methodical instructions

for performing of the calculation-graphic work

for students of the specialty 191 «Architecture and Urban Planning»

of the education program «Architecture and Urban Planning»

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Includes short theoretical data, methodical instructions, examples of the analysis of the statically determinate bar systems: the three hinged arch, the simple and the folded frames.

For students of the specialty 191 «Architecture and Urban Planning» of the education program «Architecture and Urban Planning».

Містять короткі теоретичні відомості, методичні вказівки, приклади розрахунку статично визначуваних стержневих систем: тришарнірної арки, простої та складеної рам.

Призначено для студентів спеціальності 191 «Архітектура та містобудування» освітньої програми «Архітектура та містобудування».

CONTENTS

Basic Principles

Basics of Construction Theory is one of disciplines that are forming the basis of the qualitative education of bachelors in the field of architecture. This subject is teaching as the part of the course of Structural Mechanics. Studying the discipline the student learns basics methods of analysis of building constructions under static loads. The basis of the discipline are knowledge previously received from fields of High Mathematics, Physics and Strength of Materials. Knowledge and principles that was given by Basics of Construction Theory are important during performing of course projects of Building Structures and degree designing.

Studying the discipline for future specialists-architects is divided into two parts, the first of which is teaching in the second year and the second is teaching in the third year. Methodical instructions are necessary for higher education seeker of the education program «Architecture and Urban Planning» at self-studying and during performing of the calculation-graphic work on the discipline Basics of Construction Theory – II Part. Performing of the work is based on successful assimilation of educational material that is presented in sources [1-5].

Short theoretical data that acquaint students with such bar structures as: the three hinged arch, the simple and the folded frames, their properties and peculiarities of analysis are appertaining to methodical instructions. The next part presents the example of the performing of the task that shows the order of the static analysis of these systems.

The calculation-graphic work is made out tidily on the one side of A4 format papers manually or electronically. The first page of the work is the title page. The given task is inserting into the work and the work is fastening.

Short Theoretical Data

Arches have a variety of application when designing of bridges, underground engineering constructions and roof structures. One of kinds of arched structures is a three hinged arch (Fig. 1). It represents a plane geometrically stable system that is composed of two curvilinear disks that are connected between each other and a foundation in pairs by three hinges that do not lie on the same straight line.

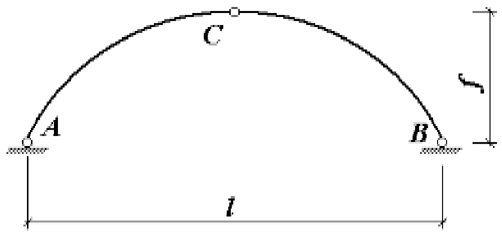


fig. 1

The arch height is called the rise of the arch *f*, the horizontal distance between supports – the span of the arch *l*. Hinged supports *A* and *B* are referred to as foot supports and the hinge *C* between members of a arch – as the top that is usually located in the centre of the span.

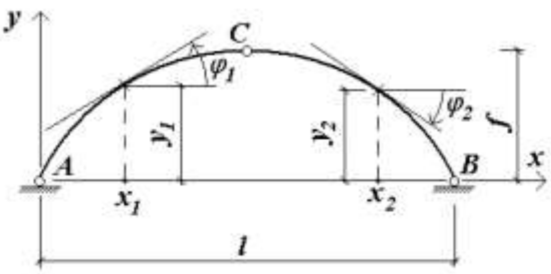


fig. 2

The axial line (the centre line) of a arch is described by the curvilinear function. The origin of the coordinate system corresponds to the left support (Fig. 2). Equations of the axial line of a arch and the angle φ (the cross-sectional dip) formed by the tangent to the centre line of a arch and the horizontal axis *x* are:

- for the parabola:

$$y = \frac{4f}{l^2} x(l - x), \ \varphi = arctg\left(\frac{4f}{l^2}(l - 2x)\right), \tag{1}$$

- for the sinusoid:

$$y = f \sin\frac{\pi x}{l}, \ \varphi = arctg\left(\frac{\pi f}{l}\cos\frac{\pi x}{l}\right), \tag{2}$$

where x denotes a coordinate of the center of the section on the horizontal axis; y denotes a distance between the center of the section and the axis of abscissa.

Under the action of only vertical loads on a three hinged arch (Fig. 3) in its two foot supports unlike in a simple beam also horizontal reactions develop that are equal in magnitude and called the horizontal thrust (H_A=H_B=H). To determine vertical support reactions (V_A, V_B) in a arch enough to write equations of the algebraic sum of moments about support dots for the entire system as a free body; to determine horizontal reactions (H_A, H_B) – additional equations of the algebraic sum of moments about the internal hinge for left and right parts (portions) of a arch.

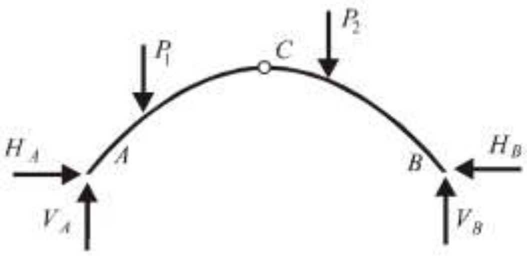


fig. 3

To determine internal forces on the i-th cross section of a arch: the bending moment M_i, the shear Q_i and the axial forces N_i equilibrium equations for free bodies of left or right parts of a arch from the cross section (Fig. 4) should be written:

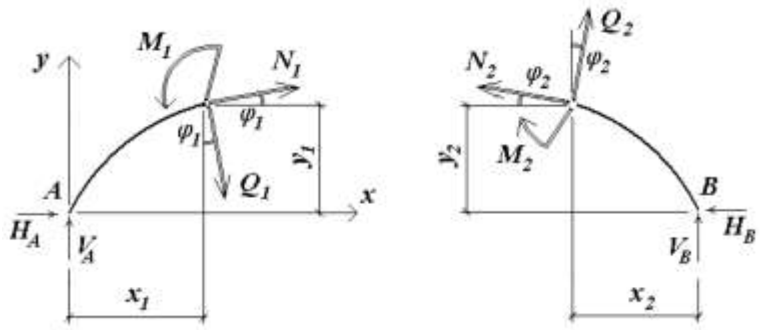


fig. 4

To analyze a three hinged arch conveniently to consider the corresponding simply supported beam that has the same span and is under the action of the same load. In this case internal forces can be calculated by using transformed formulas:

$$M_i^a = M_i^b - Hy_i ,$$

$$Q_i^a = Q_i^b \cos\varphi_i - H\sin\varphi_i , \qquad (3)$$

$$N_i^a = -Q_i^b \sin\varphi_i - H\cos\varphi_i ,$$

where M_i^b, Q_i^b denote values of bending moments and shear internal forces at the corresponding section of the corresponding statically determinate beam (Fig. 5):

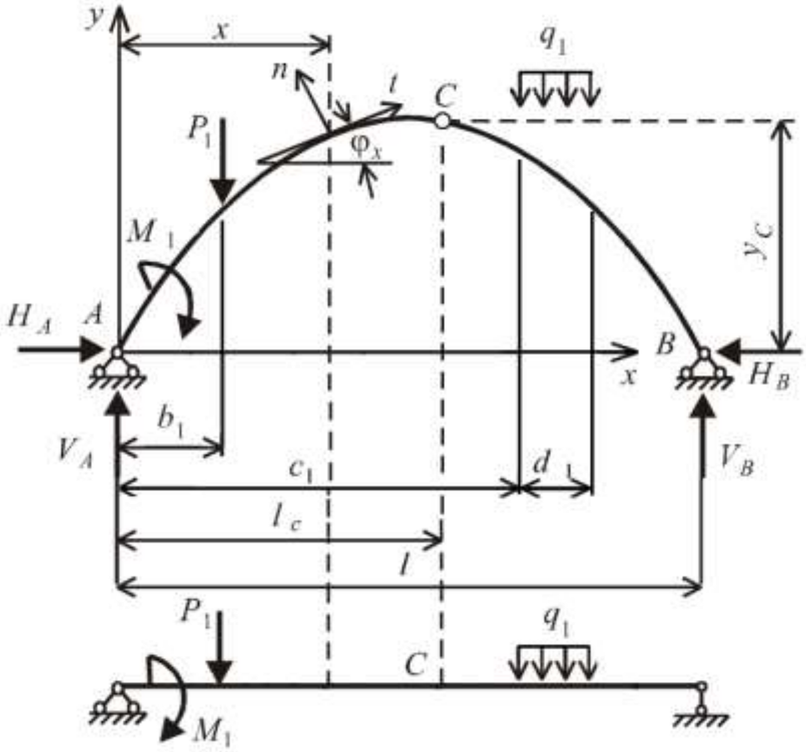
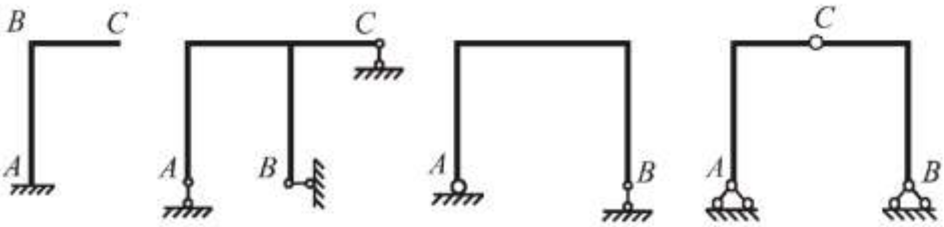


fig. 5

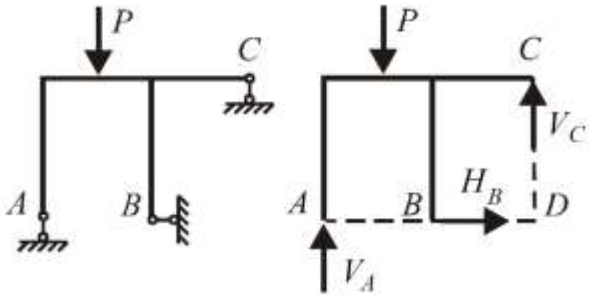
Structures consisting of bars that are connected between each other in joints, some of them are rigid, are called frames (rigid frames). According to the quantity of stages of disks connecting during the kinematic analysis frames can be divided as: simple frames (the geometric construction analysis is developed by one stage) and folded frames (the geometric construction analysis is developed by two or more stages). Simple frames according to the geometric construction rule with the foundation have the following types: overhanging (cantilever) (Fig. 6 *a*), simply supported (Fig. 6 *b*, *c*), three hinged (Fig. 6 *d*).



a *b* *c* *d*

fig. 6

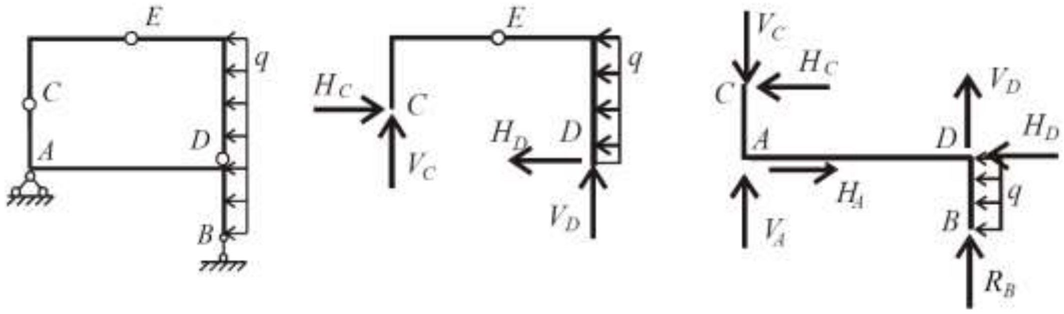
Thee support reactions of a simply supported frame (Fig. 7 *a*) are determined using three equilibrium equations of the entire frame as a free body (Fig. 7 *b*).



a *b*

fig. 7

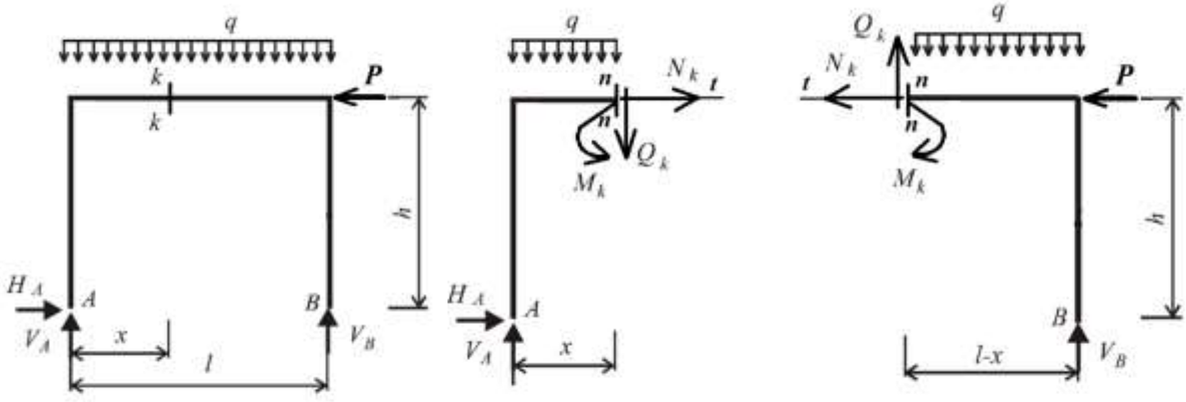
Folded frames (Fig. 8 *a*) represent few simple frames that rely one on one. Folded frames can be: multispan and multistory. In that case the analysis consists not only in calculation of support reactions but in determination of restraint forces (forces of internal disks interaction between each other) too. The calculation order should be the reverse to the geometric construction analysis of a frame scilicet from the upper «storey» (Fig. 8*b*) to the lower «storey» (Fig. 8*c*). «Support» reactions of the upper «storey» are applying on the lower with the reverse sign as the external loads.



a *b* *c*

fig. 8

At every section *k* of a frame (Fig. 9 *a*) exist three kind of internal forces too. Their values can be computed by utilizing the algebraic sum of moments of all the external loads about the centroid of the cross section (to calculate the bending moment M_k), the algebraic sum of projections of all external loads in the direction of the normal *n* to the axis of the member (to calculate the shear force Q_k) or of the tangent *t* to the axis of the member of a frame (to calculate the axial force N_k) from the either side (left or right) of the section (Fig. 9 *b*, *c*).



a *b* *c*

fig. 9

Respectively expressions for calculating internal forces are:

$$M_k = \sum M_k^l \text{ or } M_k = \sum M_k^r;$$
$$Q_k = \sum F_n^l \text{ or } Q_k = \sum F_n^r; \qquad (4)$$
$$N_k = \sum F_t^l \text{ or } N_k = \sum F_t^r.$$

During construction of internal forces diagrams ordinates of the diagram *M* are depicted on the tension side of the member herewith the sign convention is not regulated. The diagram *Q* has the positive sign if the summary shear force at the section makes the member rotate clockwise; the diagram *N* is positive if the axial force makes the member of a frame in tensile state.

To make sure that values of internal forces are calculated correctly two check necessary to perform. First of them, the check of the equilibrium of free bodies of frame's joints, consist in the check of equilibrium conditions of each joint isolated from the system after applying taken from diagrams internal forces to member's ends. The sum of projections in *x* and *y* directions and the sum of moments about an isolated joint should be equal to zero. Also the relationship between *M* and *Q* diagrams should be observed what the second check consist in. Whereas the diagram *Q* is the derivative of the diagram *M* on segments where the bending moments diagram is linear the value of the shear force must be equal to the tangent of the angle between the straight line representing diagram *M* and the axis of the member. If the diagram *M* deflects from the member's axis clockwise then the diagram *Q* has the positive sign. For verifying on segments with the curvilinear bending moments diagram is necessary to consider the isolated member of the frame applying on it the distributed load, moments at its ends and unknown shear forces. In the presence of zero on the diagram *Q* the extreme should be observed on the diagram *M* at that section.

To evaluate a displacement in bar systems usually Mohr's method (unit load method) is using. At first in that case is necessary to consider the real system (freight state) and to construct internal forces diagrams *M*, *Q* and *N* due to external loads. After that is required to create the virtual system removing all external loads and applying the unit load that corresponds to the desired displacement. Herewith, the unit horizontal load corresponds to the horizontal displacement; the unit vertical load – to the vertical displacement; the bending moment equal *1m* – to the angle of the rotation of the section; the fracture angle of the bar axis at the hinge can be induced by the simultaneous application of two unit bending moments to the bar from both sides of the hinge opposite in direction; the convergence of points can be generated as the result of application of two unit loads applied along the straight line connected these points towards each other. Whereas deformation of frames are mainly caused by bending moments then is enough to construct only the bending moments diagram M_1 during consideration of the virtual system and to calculate the displacement Δ_{1P} using the simplified Maxwell-Mohr's formula:

$$\Delta_{1P} = \sum \int_l \frac{M_1 \cdot M}{EI} dx, \tag{5}$$

where *l* is the length of the integration segment.

The integral in the expression (5) can be evaluated using numerical integration methods. One of them is the Vereshchagin's rule:

$$\int_0^l M \cdot M_1 dx = A \cdot y_i, \tag{6}$$

where *A* is the area of the diagram *M* at the segment, y_i is the ordinate of the diagram M_1 under the diagram *M* centre of the gravity.

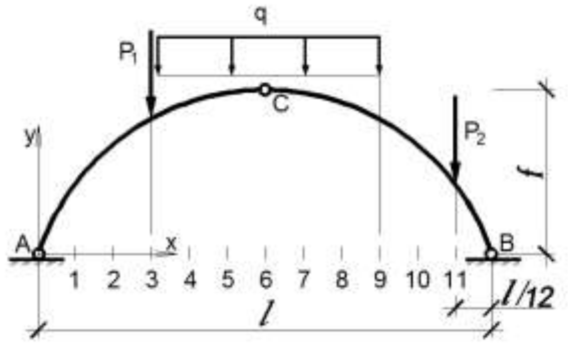
The second method of multiplication of diagrams is using of Simpson-Kornouhov's formula:

$$\int_0^l M \cdot M_1 dx = \frac{l}{6}\left[a \cdot a_1 + 4 \cdot b \cdot b_1 + c \cdot c_1\right], \tag{7}$$

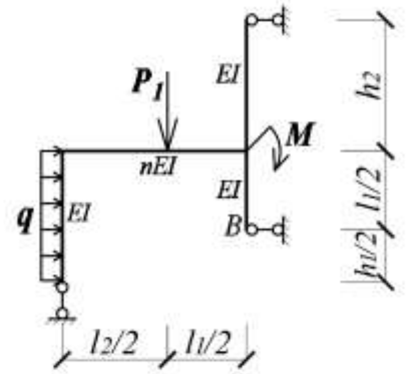
where *a*, *b*, *c* and a_1, b_1, c_1 are values at ends and at the middle of the integration area of diagrams *M* and M_1 respectively.

The Example of the Individual Task

(the front side of the task)

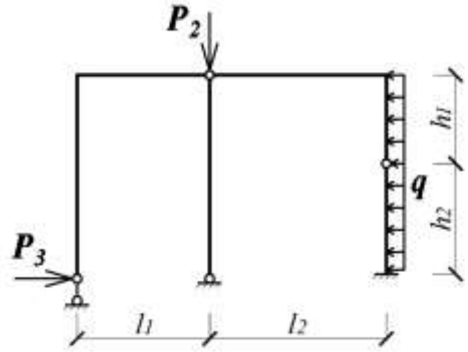


l, m	f, m	P_1, kN	P_2, kN	q, kN/m	the axis of the arch
8,4	**6,0**	**15**	**14**	**2,5**	**the parabola**
7,2	2,5	4,0	5,0	2,0	the sinusoid
9,6	4,0	5,0	6,0	2,4	the parabola
10,8	4,5	4,0	6,0	3,0	the sinusoid
12,0	5,0	5,0	8,0	3,2	the parabola



Evaluate the vertical displacement of the point B

№	l_1, m	l_2, m	h_1, m	h_2, m	q, kN/m	P_1, kN	P_2, kN	P_3, kN	M, kNm	n
1	**6,0**	**8,0**	**4,0**	**5,0**	**2,0**	**7**	**3**	**7**	**3**	**2**
2	3,6	4,0	3,0	2,4	1,6	4	7	5	6	1,5
3	2,0	3,4	2,4	4,0	1,8	15	16	18	7	2,5
4	4,0	2,4	3,8	3,0	1,4	3	8	5	8	3
5	2,4	3,2	3,6	2,0	2,2	4	7	8	9	1,5



Underlined values in the table mean data that are defined to every student by the university teacher for further calculations.

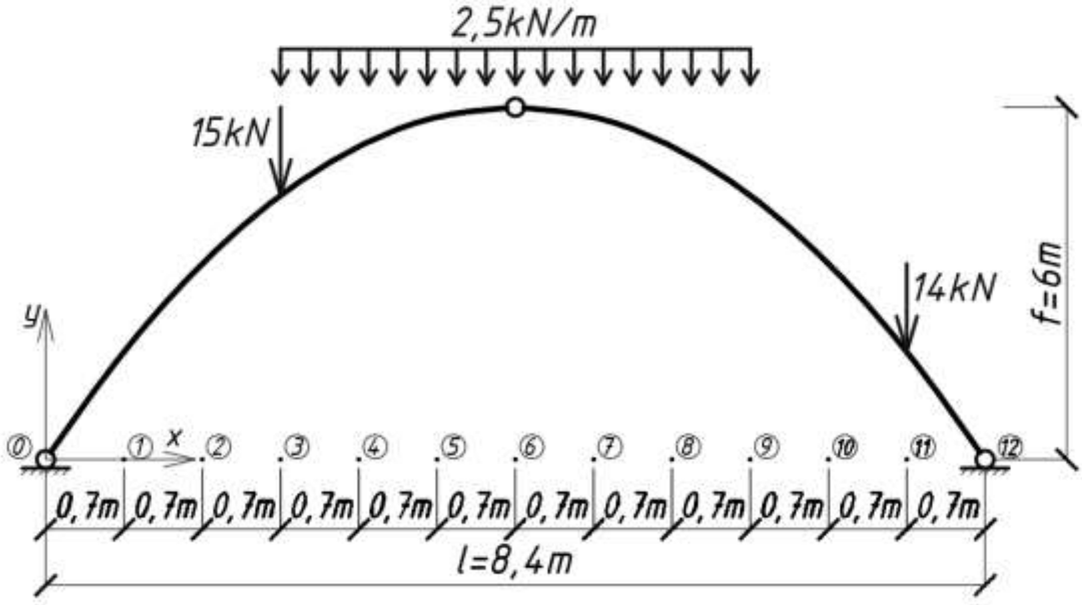
(the back side of the task)

THE ANALYSIS OF THE THREE HINGED ARCH AND STATICALLY DETERMINATE FRAMES
The analysis of the three hinged arch: 1. To perform the kinematic analysis. 2. To determine support reactions. To verify the equilibrium of the arch. 3. To determine internal forces: bending moments, shear and axial forces at sections of the arch with the step of l/12. 3. To construct internal forces diagrams of the arch using calculated values. 4. To verify calculated internal forces at sections **3** and **10** by the projection on axes. **The analysis of the simple and the folded frames**: 1. To perform the kinematic analysis. 2. To determine support reactions and restraint forces, to perform checks. 3. To construct diagrams of bending moments, shear and axial forces. 4. To perform checks of diagrams (the check of the equilibrium of free bodies of frame's joints, the check of the relationship between *M* and *Q* diagrams). **For the simple frame**: 6. To create the virtual system for determination of the displacement. 7. To determine support reactions of the virtual system and to construct the diagram of bending moments. 8. To calculate the desired displacement using Maxwell-Mohr's formula (to prepare diagrams for numerical integrating and to evaluate the displacement using Vereshchagin's and Simpson-Kornouhov's rules).

Underlined values mean sections that are defined by the university teacher to perform the check when analysing of the three hinged arch.

The Example of the Performing of the Individual Task

The Analysis of the Three Hinged Arch

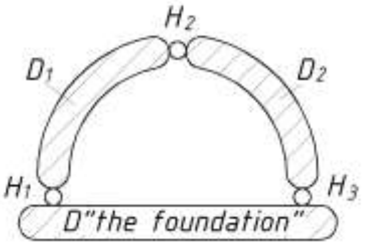


The axis of the arch is the parabola (1):

$$y = \frac{4 \cdot 6}{(8,4)^2} x(8,4 - x)\,,\ \varphi = arctg\left(\frac{4 \cdot 6}{(8,4)^2}(8,4 - 2x)\right).$$

1. <u>The kinematic analysis.</u>

1.1 The quantitative stage:



$D = 3$,
$J = 0$,
$F = 0$,
$H = 3$,
$L = 0$.

$$G = 3 \cdot D + 2 \cdot J - 3 \cdot F - 2 \cdot H - L - 3 =$$
$$= 3 \cdot 3 + 2 \cdot 0 - 3 \cdot 0 - 2 \cdot 3 - 0 - 3 = 9 - 6 - 3 = 0\,.$$

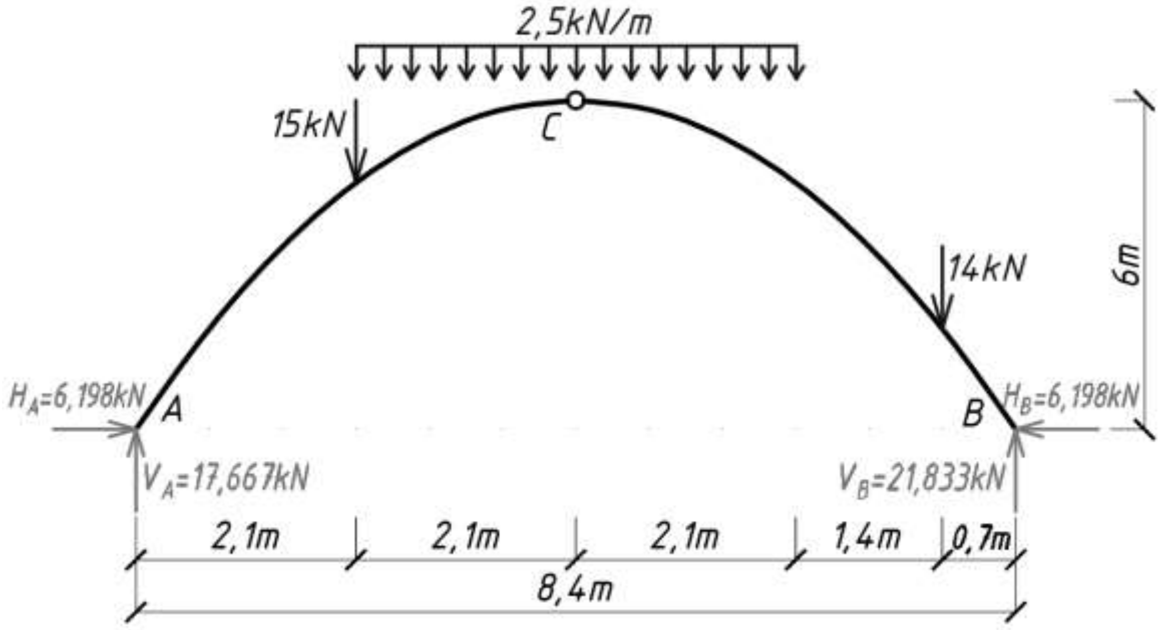
The system has the minimally required quantity of restraints.

1.2 The geometric construction analysis:

$$\frac{D_1 + D_2 + D\text{"the foundation"}}{H_1,\, H_2,\, H_3} = Д\, I$$ (the hinge triangle connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>



$\sum M_A = 0$: $15{\cdot}2{,}1{+}2{,}5{\cdot}4{,}2{\cdot}4{,}2{+}14\cdot 7{,}7{-}V_B\cdot 8{,}4=0$;

$$V_B = \frac{183{,}4}{8{,}4} = 21{,}833 kN.$$

$\sum M_B = 0$: $V_B\cdot 8{,}4 - 15{\cdot}6{,}3{-}2{,}5{\cdot}4{,}2{\cdot}4{,}2{-}14\cdot 0{,}7=0$;

$$V_A = \frac{148{,}4}{8{,}4} = 17{,}667 kN.$$

The check of vertical reactions:

$\sum F_y = 17{,}667 - 15{-}2{,}5{\cdot}4{,}2 - 14 + 21{,}833 = 39{,}5 - 39{,}5 = 0$.

$\sum M_C^l = 0$: $17{,}667{\cdot}4{,}2{-}15\cdot 2{,}1{-}2{,}5\cdot 2{,}1\cdot 1{,}05{-}H_A\cdot 6=0$;

$$H_A = \frac{37{,}189}{6} = 6{,}198 kN.$$

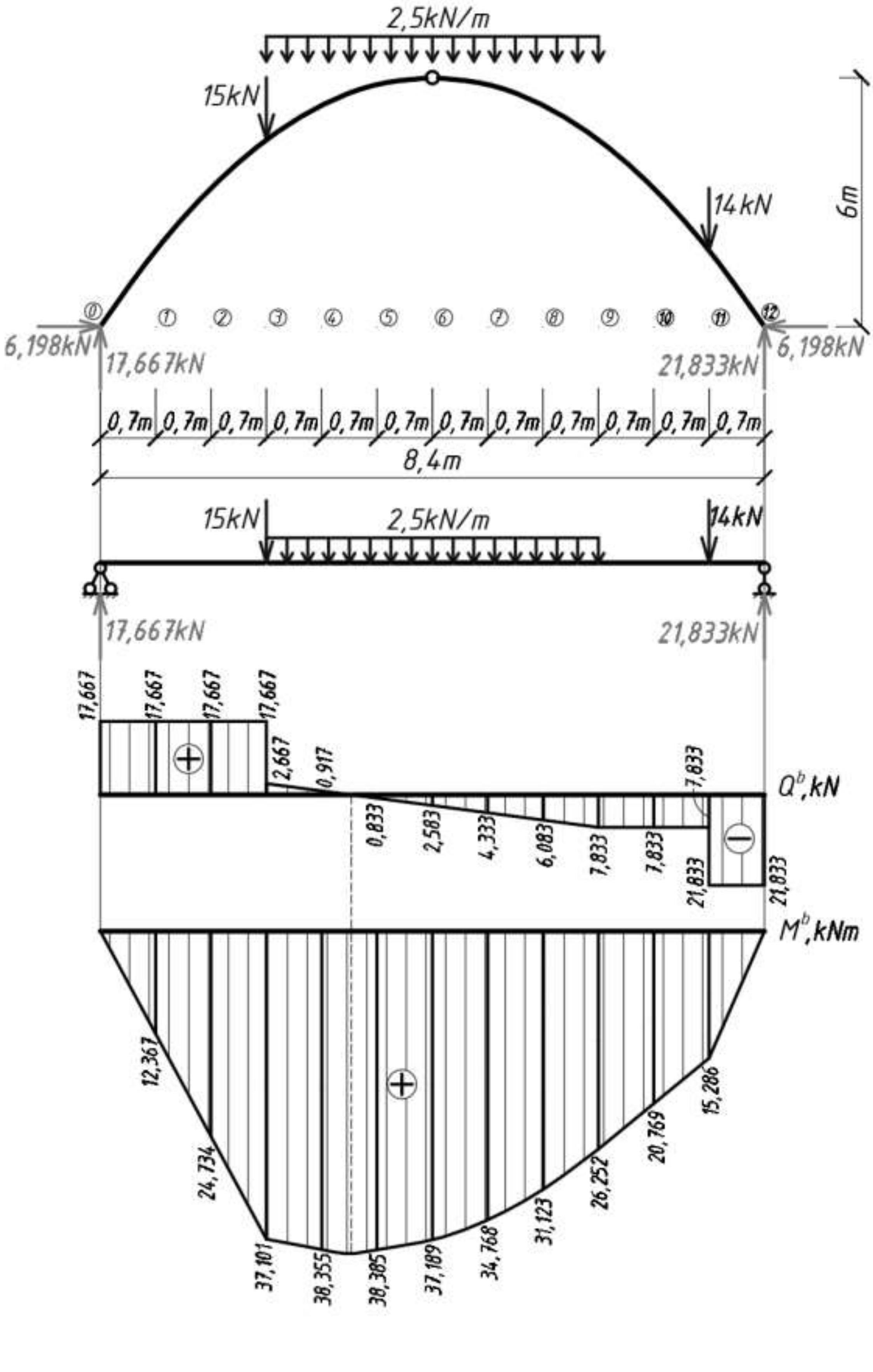
$\sum M_C^r = 0$: $2{,}5\cdot 2{,}1\cdot 1{,}05 + 14\cdot 3{,}5 - 21{,}833{\cdot}4{,}2{+}H_B\cdot 6=0$;

$$H_B = \frac{37{,}186}{6} = 6{,}198 kN.$$

The check of horizontal reactions:

$\sum F_x = 6{,}198 - 6{,}198 = 0$.

3. Depiction of diagrams of the corresponding beam.



$Q_0^b = +17,667 kN$;
$Q_1^b = +17,667 kN$;
$Q_2^b = +17,667 kN$;
$Q_{3(left)}^b = +17,667 kN$;
$Q_{3(right)}^b = +17,667 - 15 = 2,667 kN$;
$Q_4^b = +17,667 - 15 - 2,5 \cdot 0,7 = 0,917 kN$;
$Q_5^b = +17,667 - 15 - 2,5 \cdot (2 \cdot 0,7) = -0,833 kN$;
$Q_6^b = +17,667 - 15 - 2,5 \cdot (3 \cdot 0,7) = -2,583 kN$;
$Q_7^b = +17,667 - 15 - 2,5 \cdot (4 \cdot 0,7) = -4,333 kN$;
$Q_8^b = +17,667 - 15 - 2,5 \cdot (5 \cdot 0,7) = -6,083 kN$;
$Q_9^b = +17,667 - 15 - 2,5 \cdot (6 \cdot 0,7) = -7,833 kN$;
$Q_{10}^b = +17,667 - 15 - 2,5 \cdot (6 \cdot 0,7) = -7,833 kN$;
$Q_{11(left)}^b = +17,667 - 15 - 2,5 \cdot (6 \cdot 0,7) = -7,833 kN$;
$Q_{11(right)}^b = +17,667 - 15 - 2,5 \cdot (6 \cdot 0,7) - 14 = -21,833 kN$;
$Q_{12}^b = -21,833 kN$.
$M_0^b = 0$;
$M_1^b = +17,667 \cdot 0,7 = 12,367 kNm$;
$M_2^b = +17,667 \cdot (2 \cdot 0,7) = 24,734 kNm$;
$M_3^b = +17,667 \cdot (3 \cdot 0,7) = 37,101 kNm$;
$M_4^b = +17,667 \cdot (4 \cdot 0,7) - 15 \cdot 0,7 - 2,5 \cdot 0,7 \cdot 0,35 = 38,355 kNm$;
$M_5^b = +17,667 \cdot (5 \cdot 0,7) - 15 \cdot (2 \cdot 0,7) - 2,5 \cdot (2 \cdot 0,7) \cdot 0,7 = 38,385 kNm$;
$M_6^b = +17,667 \cdot (6 \cdot 0,7) - 15 \cdot (3 \cdot 0,7) - 2,5 \cdot (3 \cdot 0,7) \cdot 1,05 = 37,189 kNm$;
$M_7^b = +17,667 \cdot (7 \cdot 0,7) - 15 \cdot (4 \cdot 0,7) - 2,5 \cdot (4 \cdot 0,7) \cdot 1,4 = 34,768 kNm$;
$M_8^b = +17,667 \cdot (8 \cdot 0,7) - 15 \cdot (5 \cdot 0,7) - 2,5 \cdot (5 \cdot 0,7) \cdot 1,75 = 31,123 kNm$;
$M_9^b = +17,667 \cdot (9 \cdot 0,7) - 15 \cdot (6 \cdot 0,7) - 2,5 \cdot (6 \cdot 0,7) \cdot 2,1 = 26,252 kNm$;
$M_{10}^b = +17,667 \cdot (10 \cdot 0,7) - 15 \cdot (7 \cdot 0,7) - 2,5 \cdot (6 \cdot 0,7) \cdot 2,8 = 20,769 kNm$;
$M_{11}^b = +17,667 \cdot (11 \cdot 0,7) - 15 \cdot (8 \cdot 0,7) - 2,5 \cdot (6 \cdot 0,7) \cdot 3,5 = 15,286 kNm$;
$M_{12}^b = +17,667 \cdot (12 \cdot 0,7) - 15 \cdot (9 \cdot 0,7) - 2,5 \cdot (6 \cdot 0,7) \cdot 4,2 - 14 \cdot 0,7 = 0$.

4. <u>Determination of internal forces at sections of the arch.</u>

	x, m	y, m	φ, deg	$\sin\varphi$	$\cos\varphi$	M^b, kNm	Q^b, kN	M^a, kNm	Q^a, kN	N^a, kN
0	0	0	70,71	0,9439	0,3303	0	17,667	0	-0,014	-18,723
1	0,7	1,833	67,22	0,922	0,3872	12,367	17,667	1,004	1,127	-18,689
2	1,4	3,333	62,3	0,8854	0,4648	24,734	17,667	4,074	2,725	-18,523
$3^{лів}$	2,1	4,5	55	0,8192	0,5736	37,101	17,667	9,21	5,054	-18,028
$3^{пр}$	2,1	4,5	55	0,8192	0,5736	37,101	2,667	9,21	-3,548	-5,739
4	2,8	5,333	43,59	0,6895	0,7243	38,355	0,917	5,299	-3,610	-5,121
5	3,5	5,833	25,45	0,4297	0,903	38,385	-0,833	2,23	-3,417	-5,238
6	4,2	6	0	0	1	37,189	-2,583	0	-2,583	-6,198
7	4,9	5,833	-25,45	-0,4297	0,903	34,768	-4,333	-1,387	-1,247	-7,459
8	5,6	5,333	-43,59	-0,6895	0,7243	31,123	-6,083	-1,933	-0,130	-8,683
9	6,3	4,5	-55	-0,8192	0,5736	26,252	-7,833	-1,639	0,586	-9,971
10	7	3,333	-62,3	-0,8854	0,4648	20,769	-7,833	0,109	1,847	-9,816
$11^{лів}$	7,7	1,833	-67,22	-0,922	0,3872	15,286	-7,833	3,923	2,681	-9,622
$11^{пр}$	7,7	1,833	-67,22	-0,922	0,3872	15,286	-21,833	3,923	-2,740	-22,530
12	8,4	0	-70,71	-0,9439	0,3303	0	-21,833	0	-1,363	-22,655

Values in the table was calculated using formulas:

$$y = \frac{4\cdot 6}{(8,4)^2}x(8,4-x);$$

$$\varphi = arctg\left(\frac{4\cdot 6}{(8,4)^2}(8,4-2x)\right);$$

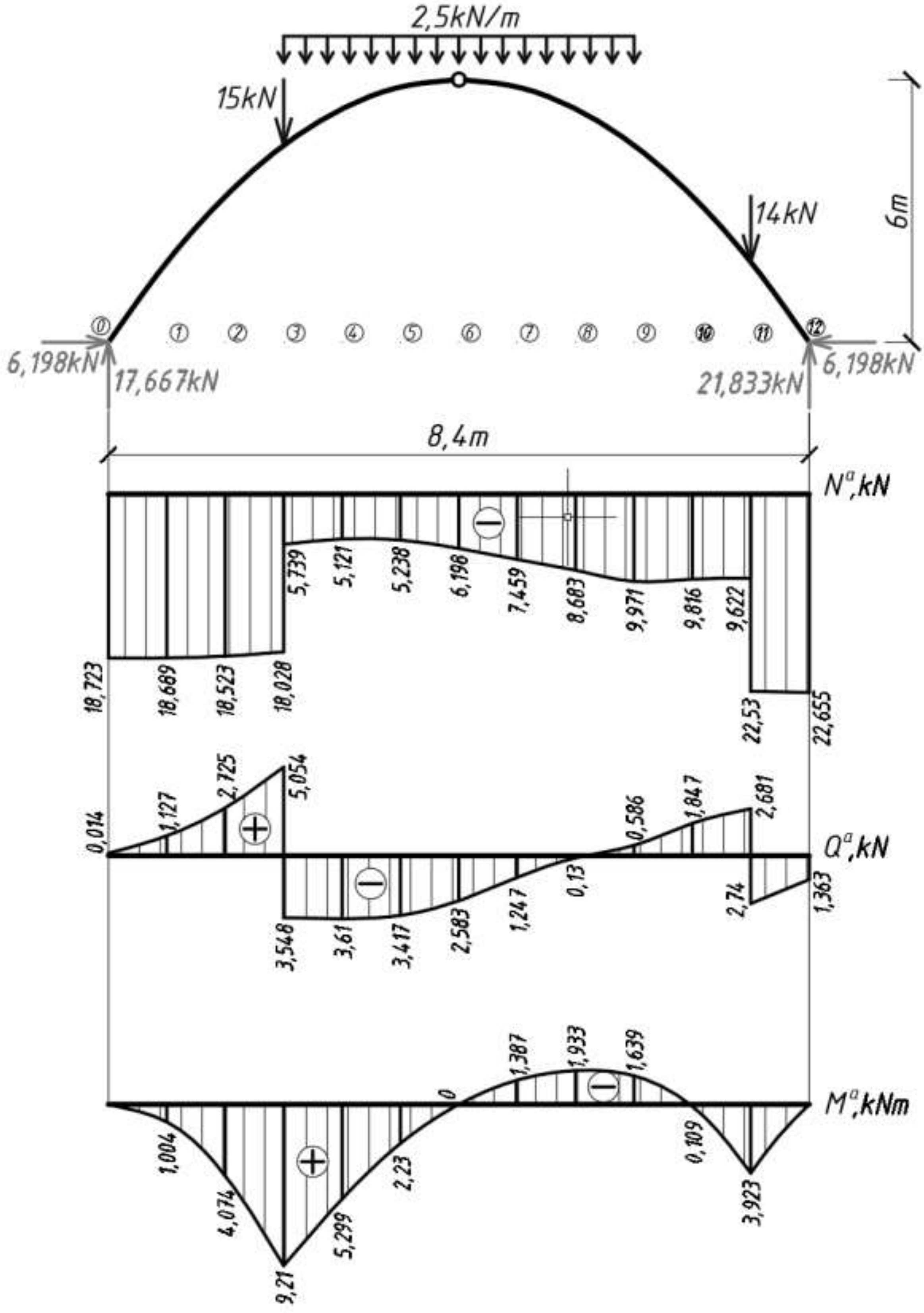
$$M^a = M^b - H\cdot y;$$

$$Q^a = Q^b\cdot\cos\varphi - H\cdot\sin\varphi;$$

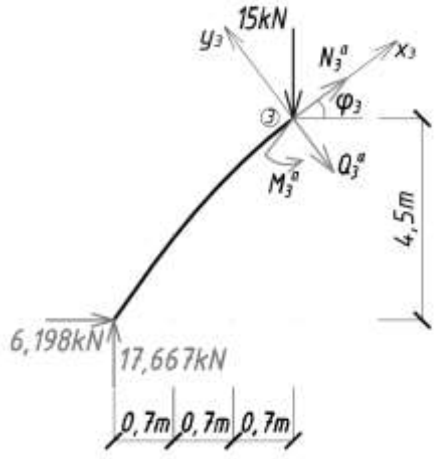
$$N^a = -(Q^b\cdot\sin\varphi + H\cdot\cos\varphi);$$

$$H = H_A = H_B = 6,198kN.$$

5. Depiction of diagrams of the arch.



6. The check of internal forces at sections of the arch.



The section 3:

$\varphi_3 = 55^\circ$

$\sin\varphi_3 = 0,8192$

$\cos\varphi_3 = 0,5736$

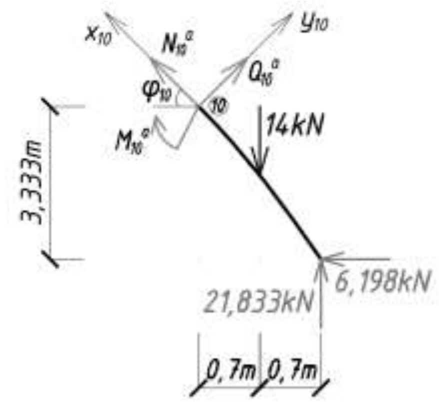
$\sum F_{x_3} = 0$: $N_3^a + 6,198 \cdot \cos\varphi_3 + 17,667 \cdot \sin\varphi_3 - 15 \cdot \sin\varphi_3 = 0$;

$N_3^a = -5,74kN$.

$\sum F_{y_3} = 0$: $-Q_3^a - 6,198 \cdot \sin\varphi_3 + 17,667 \cdot \cos\varphi_3 - 15 \cdot \cos\varphi_3 = 0$;

$Q_3^a = -3,548kN$.

$\sum M_3 = 0$: $-M_3^a - 6,198 \cdot 4,5 + 17,667 \cdot 2,1 = 0$; $M_3^a = +9,21kNm$.



The section 10:

$\varphi_{10} = 62,3^\circ$

$\sin\varphi_{10} = 0,8854$

$\cos\varphi_{10} = 0,4648$

$\sum F_{x_{10}} = 0$: $N_{10}^a + 6,198 \cdot \cos\varphi_{10} + 21,833 \cdot \sin\varphi_{10} - 14 \cdot \sin\varphi_{10} = 0$;

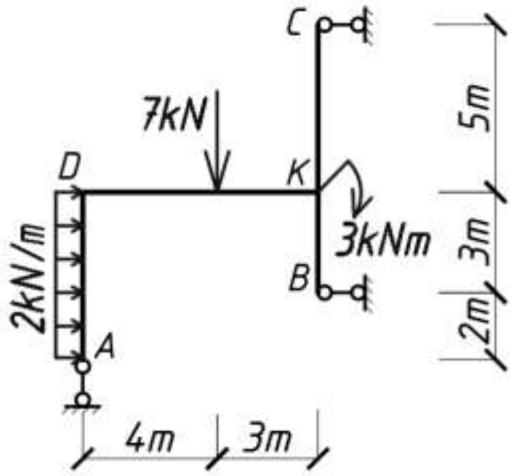
$N_{10}^a = -9,816kN$.

$\sum F_{y_{10}} = 0$: $Q_{10}^a - 6,198 \cdot \sin\varphi_{10} + 21,833 \cdot \cos\varphi_{10} - 14 \cdot \cos\varphi_{10} = 0$;

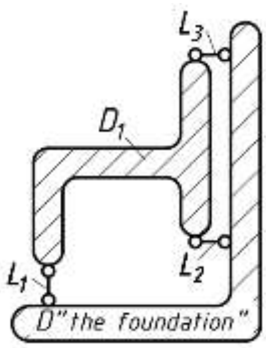
$Q_{10}^a = -1,847kN$.

$\sum M_{10} = 0$: $M_{10}^a + 6,198 \cdot 3,333 - 21,833 \cdot 1,4 + 14 \cdot 0,7 = 0$; $M_{10}^a = +0,108kNm$.

The Analysis of the Simple Frame



1. <u>The kinematic analysis.</u>



1.1 The quantitative stage:

$D = 2,\ J = 0,\ F = 0,\ H = 0,\ L = 3.$

$G = 3 \cdot D + 2 \cdot J - 3 \cdot F - 2 \cdot H - L - 3 =$

$= 3 \cdot 2 + 2 \cdot 0 - 3 \cdot 0 - 2 \cdot 0 - 3 - 3 = 6 - 6 = 0.$

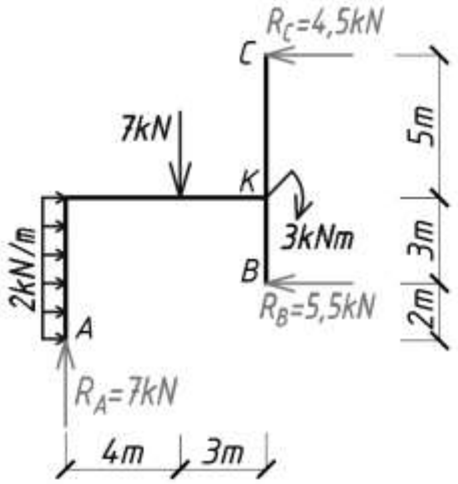
The system has the minimally required quantity of restraints.

1.2 The geometric construction analysis:

$\frac{D_1 + D"the\ foundation"}{L_1,\ L_2,\ L_3} = DI$ (Shuhov's connection).

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions.</u>



$\sum F_y = 0:\ R_A - 7 = 0;\ R_A = 7kN.$

$\sum M_C = 0:\ 7 \cdot 7 - 2 \cdot 5 \cdot 7,5 - 7 \cdot 3 + 3 - R_B \cdot 8 = 0;$

$R_B = 5,5kN.$

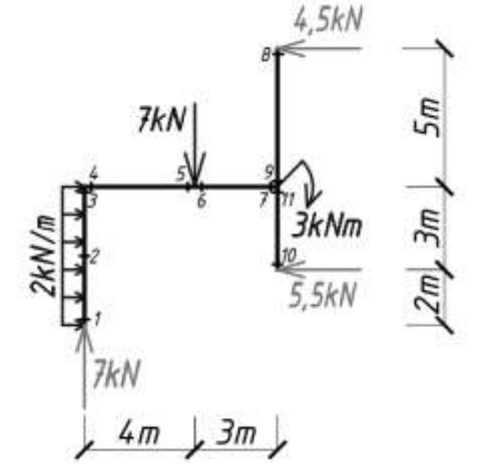
$\sum F_x = 0:\ 2 \cdot 5 - 5,5 - R_C = 0;\ R_C = 4,5kN.$

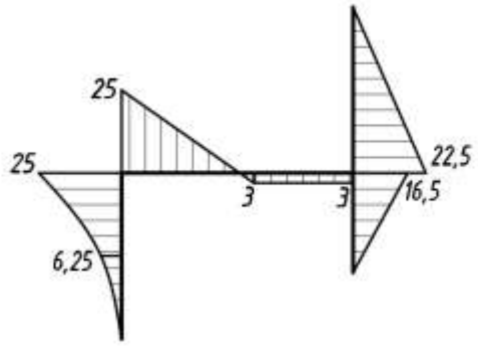
The check:

$\sum M_K = 7 \cdot 7 - 2 \cdot 5 \cdot 2,5 - 7 \cdot 3 + 5,5 \cdot 3 + 3 - 4,5 \cdot 5 =$

$= 68,5 - 68,5 = 0.$

3. <u>Depiction of internal forces diagrams.</u>





M, kNm

$M_1 = 0,$

$M_2^{low} = -2 \cdot 2,5 \cdot 1,25 = -6,25kNm,$

$M_3^{low} = -2 \cdot 5 \cdot 2,5 = -25kNm,$

$M_4^{left} = -25kNm,$

$M_5^{left} = -25 + 7 \cdot 4 = +3kNm,$

$M_6^{left} = +3kNm,$

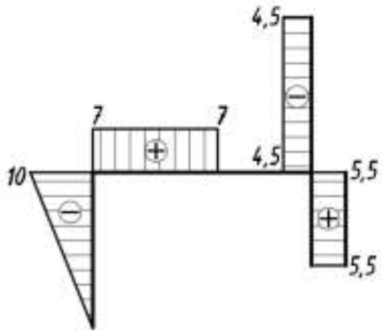
$M_7^{left} = -25 + 7 \cdot 7 - 7 \cdot 3 = +3kNm,$

$M_8 = 0,$

$M_9^{up} = -4,5 \cdot 5 = -22,5kNm,$

$M_{10} = 0,$

$M_{11}^{low} = +5,5 \cdot 3 = +16,5kNm.$



Q, kN

$Q_1 = 0,$

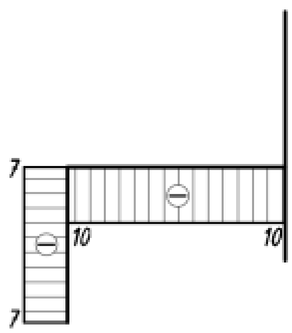
$Q_3 = -2 \cdot 5 = -10kN,$

$Q_4 = Q_5 = +7kN,$

$Q_6 = Q_7 = +7 - 7 = 0,$

$Q_8 = Q_9 = -4,5kN,$

$Q_{10} = Q_{11} = +5,5kN.$



N, kN

$N_1 = N_3 = -7kN,$

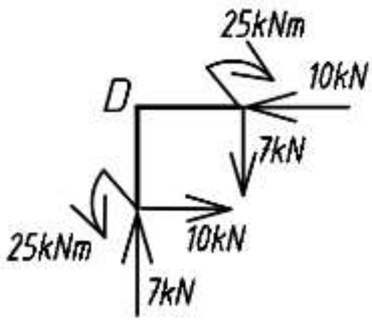
$N_4 = N_5 = -2 \cdot 5 = -10kN,$

$N_6 = N_7 = -10kN,$

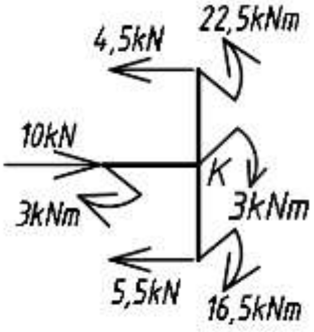
$N_8 = N_9 = 0,$

$N_{10} = N_{11} = 0.$

4. <u>The check of the equilibrium of free bodies of frame's joints.</u>



The joint *D*:
$\sum F_x = 10 - 10 = 0,$
$\sum F_y = 7 - 7 = 0,$
$\sum M_K = 25 - 25 = 0.$



The joint *K*:
$\sum F_x = 10 - 4{,}5 - 5{,}5 = 10 - 10 = 0,$
$\sum F_y = 0,$
$\sum M_K = 3 - 22{,}5 + 3 + 16{,}5 =$
$= 22{,}5 - 22{,}5 = 0.$

5. <u>The check of the relationship between *M* and *Q* diagrams.</u>

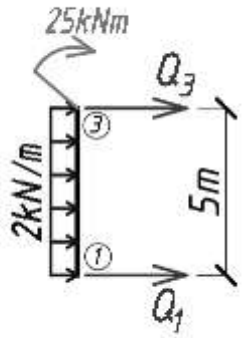
$$Q_{4-5} = +\frac{25-(-3)}{4} = +7kN\,;$$

$$Q_{6-7} = +\frac{3-3}{3} = 0\,;$$

$$Q_{8-9} = -\frac{22{,}5-0}{5} = -5{,}5kN\,;$$

$$Q_{10-11} = +\frac{16{,}5-0}{3} = -4{,}5kN\,;$$

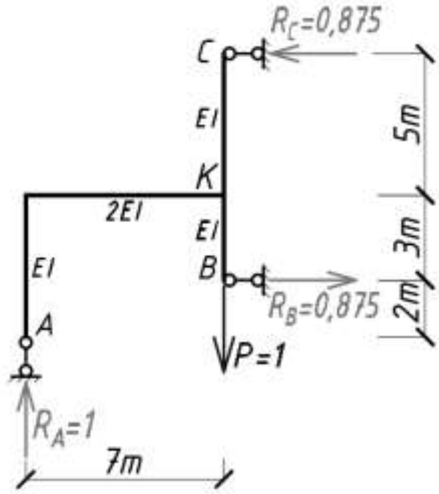
Q_{1-3}:



$\sum M_1 = 0:\; 2\cdot 5\cdot 2{,}5 + Q_3\cdot 5 + 25 = 0;$
$Q_3 = -10kN.$
$\sum M_3 = 0:\; Q_1\cdot 5 - 2\cdot 5\cdot 2{,}5 + 25 = 0;$
$Q_1 = 0.$

6. <u>Evaluation of the preset displacement (the vertical displacement of the point B).</u>

Create the virtual system corresponding to the desired displacement:

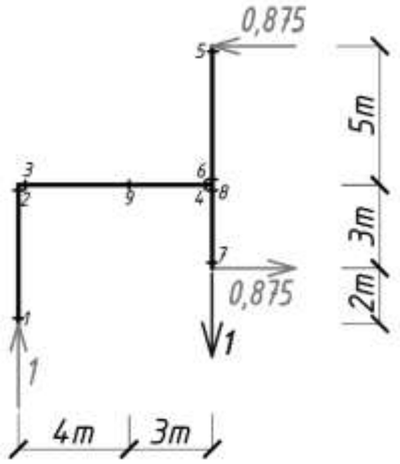


$$\sum F_y = 0:\ R_A - 1 = 0;\ R_A = 1.$$
$$\sum M_C = 0:\ 1 \cdot 7 \text{-} R_B \cdot 8 = 0;\ R_B = 0{,}875.$$
$$\sum F_x = 0:\ 0{,}875 - R_C = 0;\ R_C = 0{,}875.$$

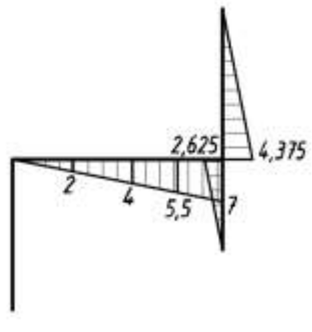
The check:

$$\sum M_K = 1 \cdot 7 \text{-} 0{,}875 \cdot 5 \text{-} 0{,}875 \cdot 3 = 7 - 7 = 0.$$

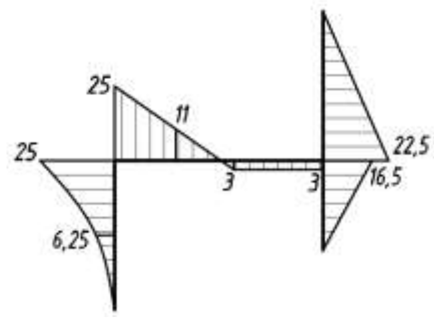
Depict the bending moments diagram due to the unit load:



$$M_1 = 0,\ M_2 = 0,\ M_3 = 0,$$
$$M_4^{left} = +1 \cdot 7 = +7m,$$
$$M_5 = 0,$$
$$M_6^{up} = -0{,}875 \cdot 5 = -4{,}375m,$$
$$M_7 = 0,$$
$$M_8^{low} = -0{,}875 \cdot 3 = -2{,}625m,$$
$$M_9^{left} = +1 \cdot 4 = +4m.$$



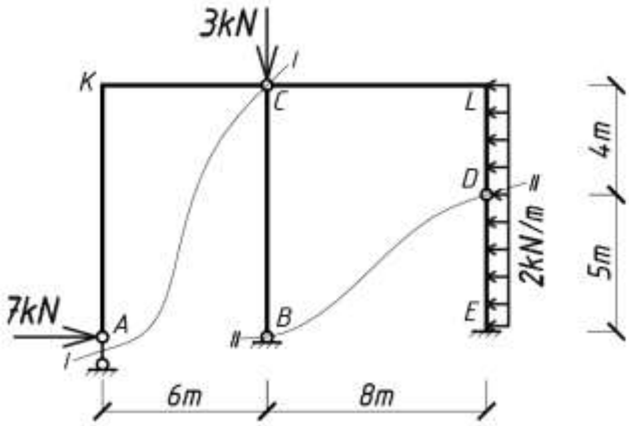
<u>M_1, m</u>



<u>M, kNm</u>

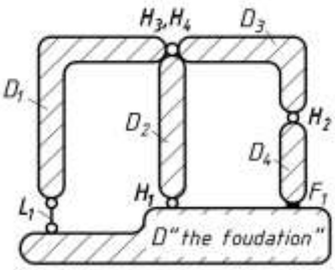
$$\Delta_{1P} = \sum \int \frac{M_1 \cdot M}{EI} dx = \frac{1}{2EI} \frac{4}{6} \left(0 \cdot 25 - 4 \cdot \frac{25-3}{2} \cdot 2 + 3 \cdot 4 \right) + \frac{1}{2EI} \left(\frac{4+7}{2} \cdot 3 \cdot 3 \right) +$$
$$+ \frac{1}{EI} \left(\frac{1}{2} \cdot 22{,}5 \cdot 5 \cdot \frac{2}{3} \cdot 4{,}375 \right) - \frac{1}{EI} \left(\frac{1}{2} \cdot 16{,}5 \cdot 3 \cdot \frac{2}{3} \cdot 2{,}625 \right) = \frac{1}{EI} (178{,}833).$$

The Analysis of the Folded Frame



1. <u>The kinematic analysis.</u>

1.1 The quantitative stage:



$D = 5,\ J = 0,\ F = 1,\ H = 4,\ L = 1.$

$G = 3 \cdot D + 2 \cdot J - 3 \cdot F - 2 \cdot H - L - 3 =$

$= 3 \cdot 5 + 2 \cdot 0 - 3 \cdot 0 - 2 \cdot 4 - 1 - 3 = 15 - 15 = 0$.

The system has the minimally required quantity of restraints.

1.2 The geometric construction analysis:

$$\frac{D''the\ foundation'' + D_4}{F_1} = D\,I \text{ (fixed support connection);}$$

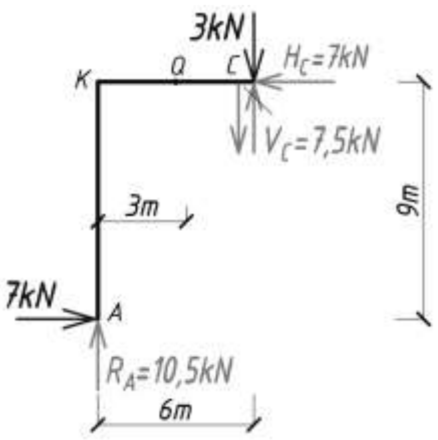
$$\frac{DI + D_2 + D_3}{H_1,\ H_2,\ H_3} = D\,II \text{ (the hinge triangle connection);}$$

$$\frac{DII + D_1}{H_4,\ L_1} = D\,III \text{ (Polonso's connection).}$$

The conclusion: the system is geometrically stable and statically determinate.

2. <u>Determination of support reactions and restraint forces.</u>

Perform the cross section I-I and consider the left portion:



$\sum M_C = 0:\ -7 \cdot 9 + R_A \cdot 6 = 0;\ R_A = 10{,}5kN$.

$\sum F_x = 0:\ 7\text{-}H_C = 0;\ H_C = 7kN$.

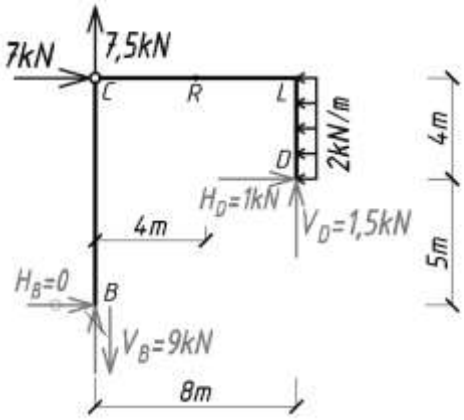
$\sum F_y = 0:\ 10{,}5 - 3 + V_C = 0;\ R_C = -7{,}5kN$.

The check:

$\sum M_Q =\ \text{-}7 \cdot 9 + 10{,}5 \cdot 3 + 3 \cdot 3 + 7{,}5 \cdot 3 =$

$= 63 - 63 = 0.$

Perform the cross section II-II and consider the left portion:



$\sum M_C^{low} = 0: \; H_B = 0$.

$\sum F_x = 0: \; 7 - H_D - 2 \cdot 4 = 0; \; H_D = 1kN$.

$\sum M_C^{r} = 0: \; 2 \cdot 4 \cdot 2 - 1 \cdot 4 - V_D \cdot 8 = 0;$

$V_D = 1{,}5kN$.

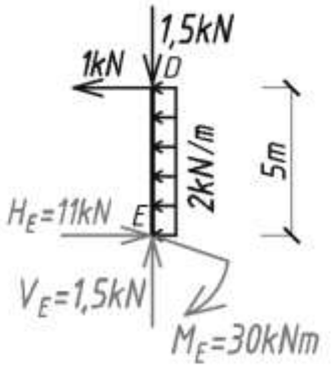
$\sum F_y = 0: 7{,}5 + 1{,}5 + V_B = 0; \; V_B = -9kN$.

The check:

$\sum M_R = -9 \cdot 4 + 7{,}5 \cdot 4 + 2 \cdot 4 \cdot 2 - 1 \cdot 4 - 1{,}5 \cdot 4 =$

$= 46 - 46 = 0.$

Evaluate the fixed support reactions:



$\sum F_x = 0: \; -1 - 2 \cdot 5 + H_E = 0; \; H_E = 11kN$.

$\sum F_y = 0: \; -1{,}5 + V_E = 0; \; V_E = 1{,}5kN$.

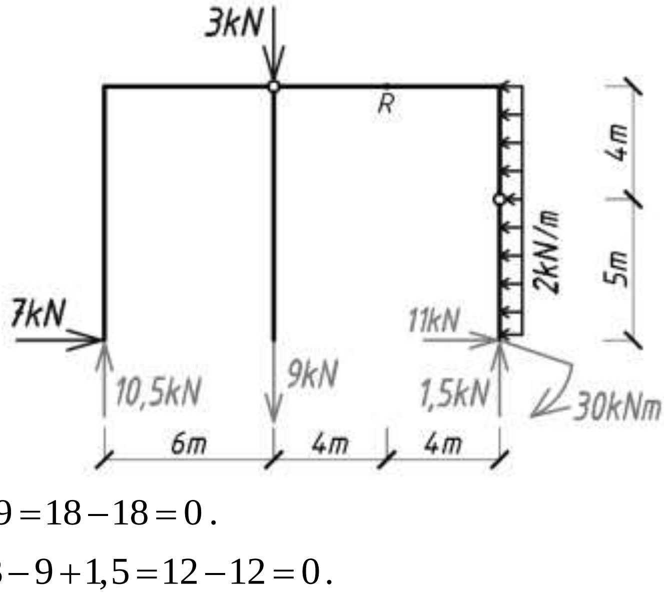
$\sum M_E = 0: \; -1 \cdot 5 - 2 \cdot 5 \cdot 2{,}5 + M_E = 0;$

$M_E = 30kNm$.

The check:

$\sum M_D = -2 \cdot 5 \cdot 2{,}5 - 11 \cdot 5 + 30 = 55 - 55 = 0.$

The check of equilibrium of the entire frame:



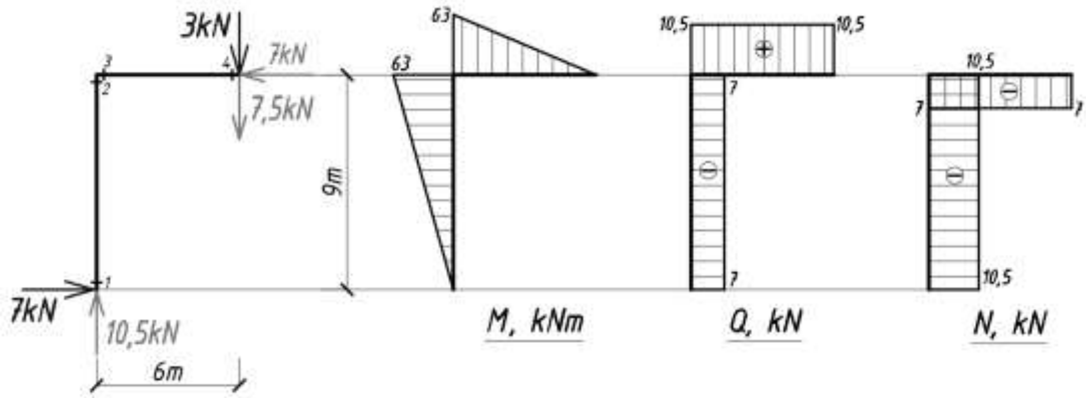
$\sum F_x = 7 + 11 - 2 \cdot 9 = 18 - 18 = 0$.

$\sum F_y = 0: \; 10{,}5 - 3 - 9 + 1{,}5 = 12 - 12 = 0$.

$\sum M_R = -7 \cdot 9 + 10{,}5 \cdot 10 - 3 \cdot 4 - 9 \cdot 4 + 2 \cdot 9 \cdot 4{,}5 - 11 \cdot 9 - 1{,}5 \cdot 4 + 30 = 216 - 216 = 0.$

3. Depiction of internal forces diagrams.

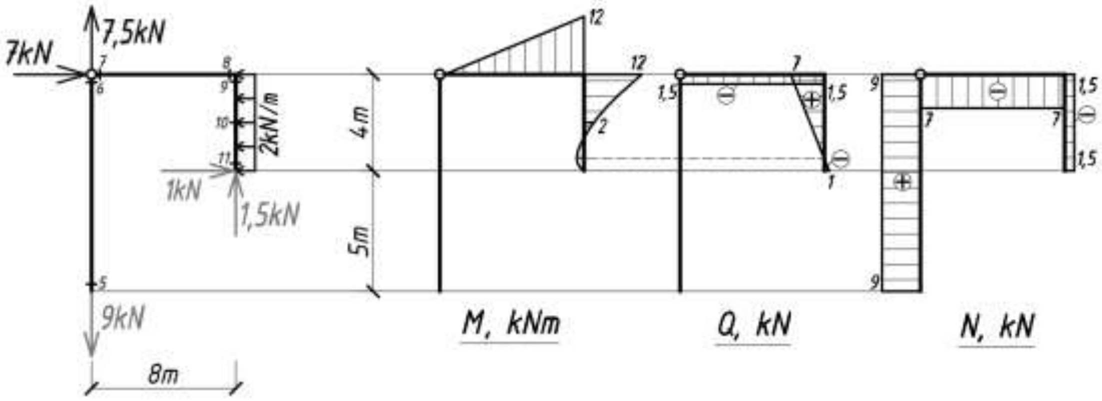
I



$M_1 = 0,\ M_2^{low} = -7 \cdot 9 = -63kNm,$ $\quad Q_1 = Q_2 = -7kN,$ $\quad N_1 = N_2 = -10{,}5kN,$

$M_4 = 0,\ M_3^{r} = +10{,}5 \cdot 6 = 63kNm.$ $\quad Q_3 = Q_4 = +10{,}5kN.$ $\quad N_3 = N_4 = -7kN.$

II



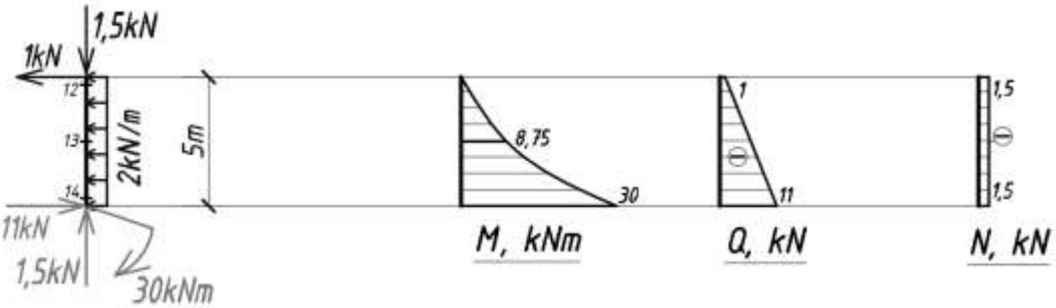
$M_5 = 0,\ M_6 = 0,\ M_{11} = 0,$ $\quad Q_5 = Q_6 = 0,$ $\quad N_5 = N_6 = +9kN,$

$M_{10}^{low} = -1 \cdot 2 + 2 \cdot 2 \cdot 1 = +2kNm,$ $\quad Q_{11} = -1\kappa H,$ $\quad N_9 = N_{11} = -1{,}5kN,$

$M_9^{low} = -1 \cdot 4 + 2 \cdot 4 \cdot 2 = +12kNm,$ $\quad Q_9 = -1 + 2 \cdot 4 = +7kN,$ $\quad N_7 = N_8 = -7kN.$

$M_8^{r} = +12kNm,\ M_7 = 0.$ $\quad Q_7 = Q_8 = -1{,}5kN.$

III

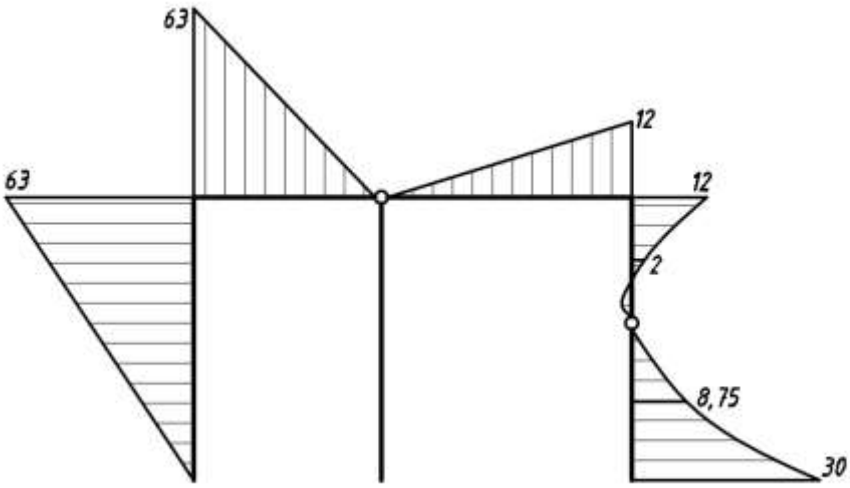


$M_{12} = 0,$ $\quad Q_{12} = -1kN,$

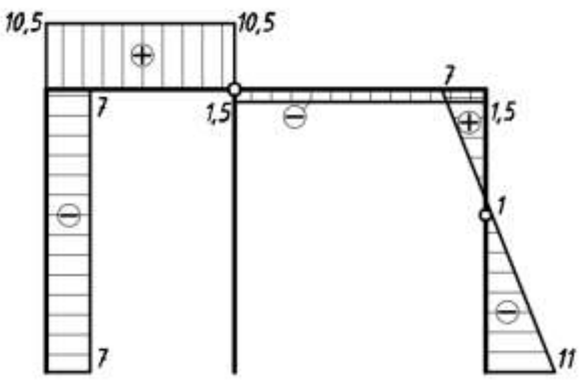
$M_{13}^{up} = -1 \cdot 2{,}5 - 2 \cdot 2{,}5 \cdot 1{,}25 = -8{,}75kNm,$ $\quad Q_{14} = -1 - 2 \cdot 5 = 11kN.$

$M_4 = -1 \cdot 5 - 2 \cdot 5 \cdot 2{,}5 = -30kNm.$ $\quad N_{12} = N_{14} = -1{,}5kN.$

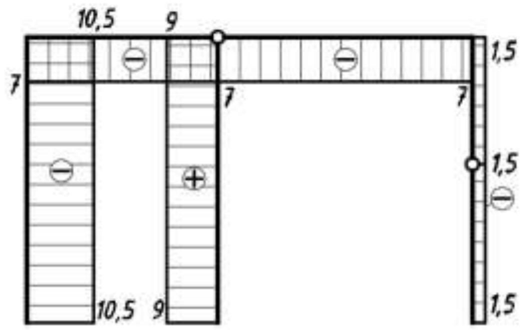
Depiction of internal forces diagrams of the entire frame



M, kNm

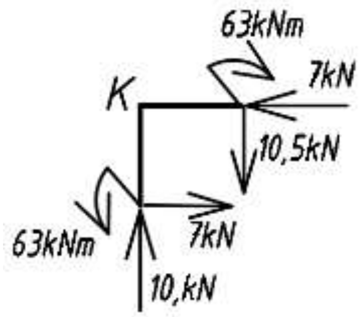


Q, kN



N, kN

4. The check of the equilibrium of free bodies of frame's joints.

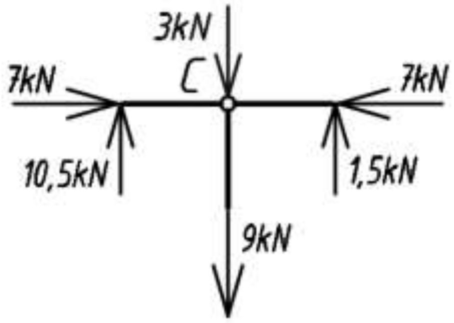


The joint K:

$\sum F_x = 7 - 7 = 0,$

$\sum F_y = 10{,}5 - 10{,}5 = 0,$

$\sum M_K = 63 - 63 = 0.$

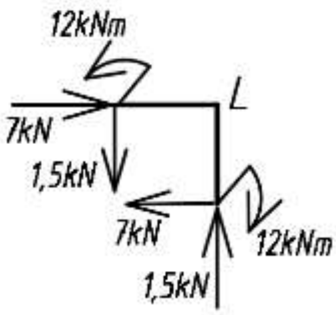


The joint C:

$\sum F_x = 7 - 7 = 0,$

$\sum F_y = 10{,}5 + 1{,}5 - 3 - 9 = 12 - 12 = 0,$

$\sum M_C = 0.$



The joint L:

$\sum F_x = 7 - 7 = 0,$

$\sum F_y = 1{,}5 - 1{,}5 = 0,$

$\sum M_L = 12 - 12 = 0.$

5. The check of the relationship between M and Q diagrams.

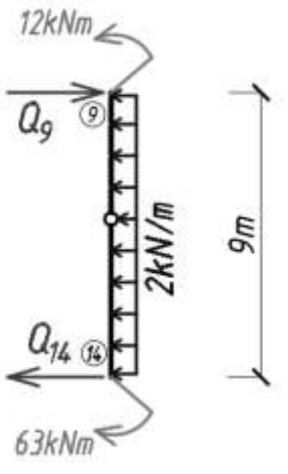
$Q_{1-2} = -\dfrac{63-0}{9} = -7kN;$

$Q_{3-4} = +\dfrac{63-0}{6} = +10{,}5kN;$

$Q_{5-6} = 0;$

$Q_{7-8} = -\dfrac{12-0}{8} = -1{,}5kN;$

Q_{9-14}:



$\sum M_9 = 0: \ -12 + 2 \cdot 9 \cdot 4{,}5 + 30 + Q_{14} \cdot 9 = 0;$

$Q_P = -11kN.$

$\sum M_{14} = 0: \ Q_9 \cdot 9 - 12 - 2 \cdot 9 \cdot 4{,}5 + 30 = 0;$

$Q_9 = +7kN.$

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for performing of the calculation-graphic work
for students of the specialty 191 «Architecture and Urban Planning»
of the education program «Architecture and Urban Planning»

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