MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
Kyiv national university of construction and architecture

THE ANALYSIS OF THE STATICALLY INDETERMINATE UNSYMMETRICAL FRAME BY USING THE FORCE METHOD

Methodical instructions
for performing of the calculation-graphic work
for students of the specialty 192 «Construction and Civil Engineering»
of the education program «Urban construction and governance»

Kyiv 2022

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Т11

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Includes short theoretical data, methodical instructions, the example of the analysis of the unsymmetrical frame by using the force method for performing of the calculation-graphic work.

For students of the specialty 192 «Construction and Civil Engineering» of the education program «Urban Construction and governance».

Містять короткі теоретичні відомості, методичні вказівки, приклад розрахунку несиметричної рами з використанням методу сил для виконання розрахунково-графічної роботи.

Призначено для студентів спеціальності 192 «Будівництво та цивільна інженерія» освітньої програми «Міське будівництво та господарство».

CONTENTS

Basic Principles

Structural Mechanics is one of fundamental disciplines that are necessary for the quality education in the field of construction whereas this science studies methods for the analysis of strength, stability and stiffness of buildings and structures. Structural Mechanics based on previously received knowledge of High Mathematics, Theoretical Mechanics and Strength of Materials and provides basic methods of analysis of building construction under static and dynamic loads. Knowledge and principles that was given by Structural Mechanics will be used during studying following courses of Building Structures and performing of course and degree projects.

Listening of the lecture course, taking of the practical course of Structural Mechanics, performing of the calculation-graphic work (CGW) are necessary to understand the principles of analysis of building structures. Methodical instructions «The Analysis of the Statically Indeterminate Unsymmetrical Frame Using the Force Method» was developed to help students to perform the CGW, to explain the algorithm and principles of the analysis of statically indeterminate systems using the force method. Also methodical materials [1-6] of Structural Mechanics that present important aspects of the analysis of structures and contain some examples of the calculation of problems can help to the student to assimilate the material better.

Methodical instructions consist of: short theoretical data that highlight the theoretical material; the example of the analysis of the statically indeterminate frame with step-by-step explanations and figures; the description of the composition of the work.

The CGW should be designed on papers of A4 format tidily. The title page is the first page of the work. The given task is inserting in the CGW after the title page and then all sheets are fastening with two staples on the left side.

Short Theoretical Data

All engineering structures and their bearing members should be motionless and their analytical models geometrically stable. Herewith, the degree of geometrically stable G is equal to or smaller than zero $G \leq 0$ is the necessary condition. From the geometric construction point of view if $G = 0$, the quantity of links that connect separate members of a structure among themselves and with a foundation is the minimum required quantity. When a system has so-called excess restraints it allows increasing it stiffness because in a case of one or some of links stoppage the destruction of a whole system does not occurs. However, there is a problem to determine internal forces because the number of restraints is greater than the number of equilibrium equations and a system is the statically indeterminate structure.

One of classic methods of the analysis of statically indeterminate structures is the force method (or the method of consistent deformation) that is especially widely used for analysis of frames. When analyzing a structure by means of the force method, enough restraints must be removed from a statically indeterminate structure to render it statically determinate that can by analyzed by using equilibrium equations. This system is a primary system of the force method and hereinafter must be analyzed. It necessarily must be statically determinate and geometrically stable. To find out how many restraints must be removed should be calculated the degrees of indeterminacy of a frame *n*. The simplified expression that is the modification of the Chebyshov's formula for frames can be written as follow:

$$n = 3 \cdot c - h, \tag{1}$$

where c is the number of closed contours, that are formed by members of a frame and the foundation (herewith the foundation must be as one rigid body); h is the number of simple hinges including supports. The hinged support is accepting as one hinge, the roller support as two hinges. The degrees of indeterminacy n are equal the degree of geometrically stable of a system G that can be calculated by using the classic Chebyshov's formula.

For example the degrees of indeterminacy n of the frame shown in the (Fig.1a) will be (Fig.1b):

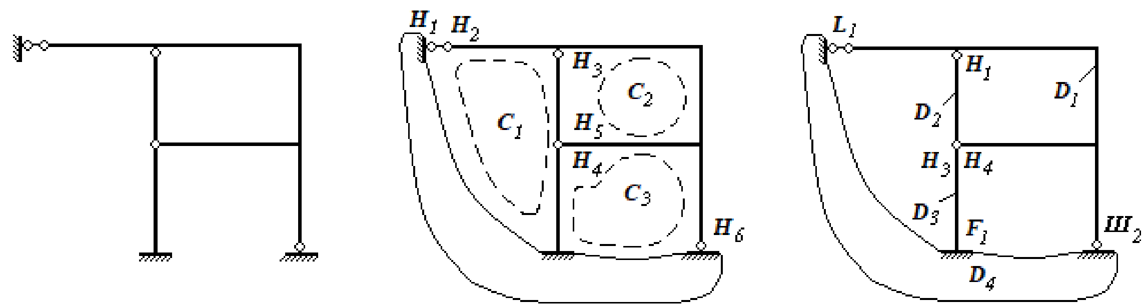
$$n = 3 \cdot 3 - 6 = 3.$$

Also can calculate the degree of geometrically stable G of this structure (Fig.1c):

$$G = 3\cdot D + 2\cdot J - 3\cdot F - 2\cdot H - L - 3 =$$
$$= 3\cdot 4 + 2\cdot 0 - 3\cdot 1 - 2\cdot 4 - 1 - 3 = 12 - 15 = 3,$$

where G is the degree of geometrically stable of the structure; D is the quantity of disks (simple rigid bodies); J is the quantity of joints; F is the quantity of fixed supports; H is the quantity of simple hinges; L is the quantity of links including supports.

Both calculated values coincide and it means that this frame is the structure with three degrees of indeterminacy or has three excess restraints.



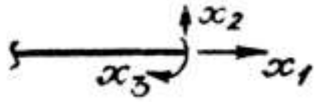
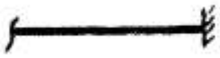
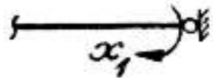
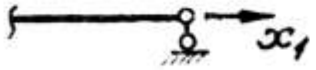
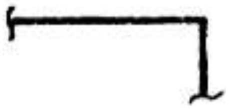
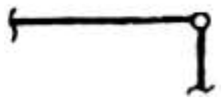
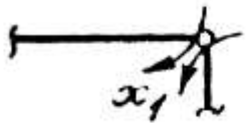
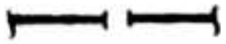
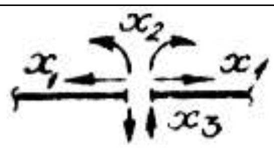
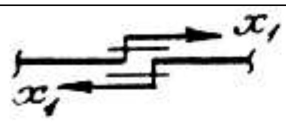
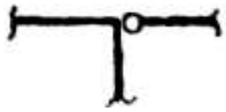
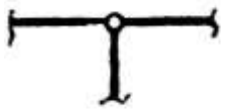
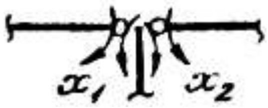
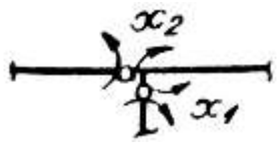
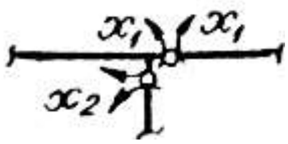
a b c

fig. 1

When choosing the primary system of the force method can be removed only such restraints that convert the system to a statically determinate structure (conditionally excess restraints) and such links can’t be removed, that change the system to the geometrically unstable or instantaneously unstable structure (absolutely necessary restraints).

Since the original frame and the primary system have different deformations and internal forces then to eliminate these divergence reactions that associated with redundant restraints in the original structure should be applied on the primary system. When removing one roller support one redundant force is applied. Adding one single hinge is equivalent to removing of one restraint herewith two equal in magnitude but opposite in direction bending moments are applied from both sides of the hinge. The cross section of the member of a frame allows removing three restraints therefore three reactions: axial, shear forces and the moment must be applied at the section. These reactions or internal forces are termed primary unknowns (variables) of the force method and initially have unknown values. Examples of removing of restraints and substituting for them corresponding reactions are illustrated in the table 1.

Table 1

Joints of a original frame	Joints of a primary system	Reactions that replace redundant restraints
		
		
		
		
		
		
		
		
		
		 or  or 

Several versions of a primary system of the force method can be selected for one frame. Quantity of primary unknowns for every statically indeterminate frame has a constant value and not depends from the selection version of the primary system of the force method.

Displacements that were impossible because of excess restraints can be presented in the selected primary system after removing of these restraints. To determine the displacement along the direction of one of redundant restraints for the structure with three degrees of indeterminacy initially the equation can write:

$$\delta_{11}X_1 + \delta_{12}X_2 + ... + \delta_{1n}X_n + \Delta_{1P} = 0, \tag{2}$$

where δ_{jk} is the displacement in the primary system along the direction of the reaction X_j due to the unit value of the unknown force X_k; $\delta_{jk} \cdot X_k$ and Δ_{jP} are displacements along the direction of the reaction X_j due to the unknown force X_k and of the known external load; equality to zero means that the summary displacement along the direction of the redundant restraint X_j due to the combined effect of forces $X_1, X_2, ... X_n$ and the external load must be equal to zero ($j, k = 1, 2, ... n$).

When write equations for every redundant restraint can be obtained the system with *n* equations where reactions in *n* redundant restraints presented as unknowns:

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + ... + \delta_{1n}X_n + \Delta_{1P} = 0, \\ \delta_{21}X_1 + \delta_{22}X_2 + ... + \delta_{2n}X_n + \Delta_{2P} = 0, \\ ... \quad ... \quad ... \quad ... \quad ... \quad ... \\ \delta_{n1}X_1 + \delta_{n2}X_2 + ... + \delta_{nn}X_n + \Delta_{nP} = 0. \end{cases} \tag{3}$$

When displacements along directions of excess restraints in the primary system of the force method are equal to zero the stress-strain state of the statically determinate system is equivalent to the stress-strain state of the original statically indeterminate system.

Flexibility coefficients δ_{jk} and free terms Δ_{jP} of the system of equations can be calculated by using the Maxwell-Mohr's formula:

$$\delta_{jk} = \sum_{j=1}^{n} \int_l \frac{M_j \cdot M_k}{EI} dx, \; (j, k = 1, 2, ... n), \tag{4}$$

$$\Delta_{jP} = \sum_{j=1}^{n} \int_l \frac{M_j \cdot M_P}{EI} dx, \; (j = 1, 2, ... n), \tag{5}$$

where M_j, M_k, M_P are bending moments in members of the primary system due to the unit value of the force X_j, X_k and the external load; EI is the bending stiffness of members of the frame.

In order to determine values of flexibility coefficients and free terms of the system of equations first consider states of the primary system: unitary (under action of unit value of one force applied along the direction of the redundant restraint) and freight (under action of external loading). Then construct unitary and freight bending moment diagrams.

The system of canonical equations of the force method has peculiarities: firstly coefficients located on the main diagonal (main coefficients) are always greater than zero ($\delta_{jj} > 0$**)**; secondly symmetrical coefficients of the main diagonal (secondary coefficients) are identical ($\delta_{jk} = \delta_{kj}$**)**.

Value of forces of redundant restraints can be determined by solving the system of equations. After this can be evaluated internal forces of the original frame. There are two methods to do this.

First of them is termed the method of superposition. It can be used if were constructed diagrams of bending moments, shear and axial forces of unitary and freight states of the primary system. According to the principle of superposition the equations used to determine internal forces of the original statically indeterminate system (M_r, Q_r, N_r) can be written as:

$$\begin{aligned} M_r &= M_1 \cdot X_1 + M_2 \cdot X_2 + \ldots + M_n \cdot X_n + M_P, \\ Q_r &= Q_1 \cdot X_1 + Q_2 \cdot X_2 + \ldots + Q_n \cdot X_n + Q_P, \\ N_r &= N_1 \cdot X_1 + N_2 \cdot X_2 + \ldots + N_n \cdot X_n + N_P. \end{aligned} \tag{6}$$

If were constructed unitary and freight bending moment diagrams only the diagram M_r can be constructed by using the method of superposition, the diagram Q_r by using the relationship between shears and bending moments and axial forces by using equilibrium equations of free bodies of joints of the frame.

According to the second method the primary system is analyzed by applying the given external loading and calculated by the system of equations reactions of redundant restraints: are determined support reactions, restraint forces and are constructed internal forces diagrams. This method is termed the static method.

To check the correctness of calculated internal forces is necessary to do the kinematic checking that is to determine the displacement that must be equal to zero. Such displacement is the displacement in the original system along the direction of one of restraints. To determine the displacement along the direction of *j*th redundant restraint can be used the Maxwell-Mohr's formula:

$$\Delta_{jr} = \sum_{j=1}^{n} \int_{l} \frac{M_j \cdot M_r}{EI} dx. \tag{7}$$

The summary displacement along directions of all redundant restraints expediently to calculate for check of the real bending moments diagram of each cross sections of the frame and to use when multiplying the summary unitary bending moment diagram M_{Σ} that was constructed by the addition of unitary diagrams:

$$M_{\Sigma} = M_1 + M_2 + ... + M_n. \tag{8}$$

The value of the displacement Δ_{jr} is usually different from zero because of the round-up of intermediate results. The relative error ε of the obtained value of Δ_{jr} should be smaller than few percent:

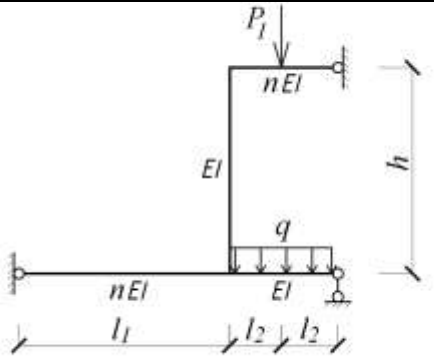
$$\varepsilon = \frac{|\Delta_{jr}|}{\Sigma_r} \cdot 100\%, \tag{9}$$

where Σ_r denotes the sum of positive numbers of the calculation.

At the end of the calculation the check of the relationship between M_r and Q_r diagrams, the check of equilibrium of free bodies of frame's joints and of the entire frame should be performed. Values and directions of support reactions can be determined from real internal forces diagrams. These checks can be not performed if some of them has already used during construction of internal forces diagrams of the original statically indeterminate frame.

The Example of the Individual Task

(the front side of the task)

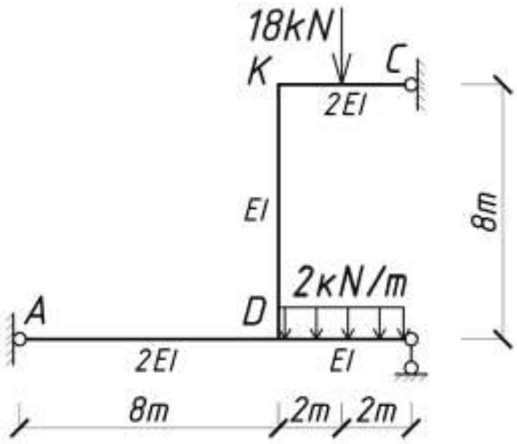


l_1	l_2	h	q	P_1	P_2	n
8	**2**	**8**	**2**	-	11	2
5	6	3	3	5	-	3
4	3	2	4	-	7	5
7	6	1	5	**18**	**-**	**2**
2	4	5	6	-	6	4

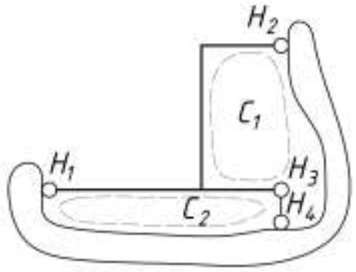
(the back side of the task)

Contents of the work
1. to determine the degrees of indeterminacy of the frame; 2. to suggest three variants of the primary system; 3. for further calculations to choose one variant of the primary system and to write the system of canonical equations of the force method in general aspect; 4. to consider unitary and freight states and to construct internal forces diagrams; 5. to calculate coefficients and free terms of the system of equations and to perform the check of their values; 6. to calculate primary unknowns of the force method; 7. to construct real diagrams of bending moments M_r, shear forces Q_r and axial forces N_r. and to perform the kinematic check and the static check; 8. to perform the equilibrium conditions of free bodies of joints of the frame and the relationship between M_r and Q_r diagrams; 9. to determine support reactions and to check equilibrium conditions of the entire system.

The Example of the Analysis of the Unsymmetrical Frame by Using the Force Method



1. Determination of the degrees of indeterminacy.



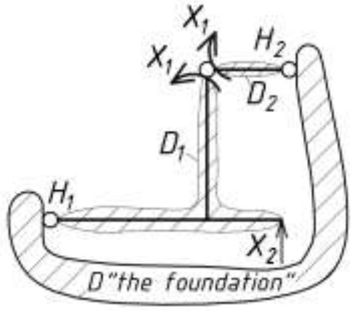
$n = 3 \cdot c - h = 3 \cdot 2 - 4 = 2$

Therefore, this frame is the structure with two degrees of indeterminacy. To analyze the problem is necessary to remove two excess restraints and to replace them by unit reactions.

2. Selection of the primary system of the force method and the system of canonical equations.

Variants of the primary system of the force method:

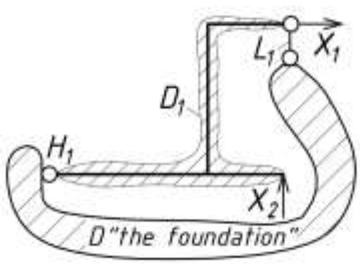
The first variant



$DI = \dfrac{D_1 + D_2 + D\text{"the foundation"}}{H_1, H_2, H_3}$

(the hinge triangle connection)

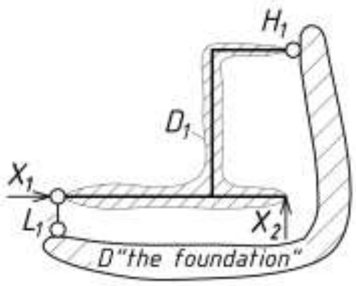
The second variant



$DI = \dfrac{D_1 + D\text{"the foundation"}}{H_1, L_1}$

(Polonso's connection)

The third variant



$DI = \dfrac{D_1 + D\text{"the foundation"}}{H_1, L_1}$

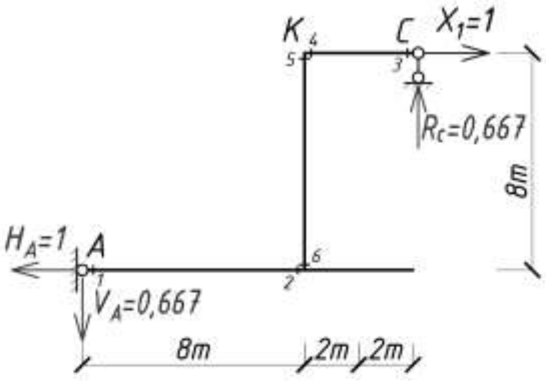
(Polonso's connection)

For further calculations choose the second variant of the primary system. The system of equations in the case of two unknowns can be written as follows:

$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \Delta_{1P} = 0, \\ \delta_{21}X_1 + \delta_{22}X_2 + \Delta_{2P} = 0. \end{cases}$$

3. <u>Determination of restraint forces (forces of internal disks interaction).</u>

3.1. The first unitary state



$\sum F_x = 0: \ -H_A + 1 = 0, \ H_A = 1;$

$\sum M_A = 0: \ 1 \cdot 8 \text{-} R_C \cdot 12 = 0,$

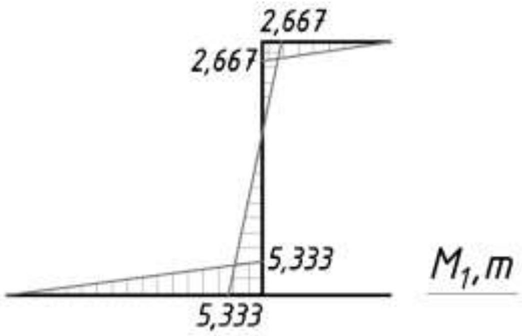
$R_C = 0,667;$

$\sum F_y = 0: \ -V_A + 0,667 = 0, \ V_A = 0,667;$

The check:

$\sum M_K = 1 \cdot 8 \text{-} 0,667 \cdot 8 - 0,667 \cdot 4 =$

$= 8 - 8,004 = -0,004 \approx 0.$



$M_1 = 0;$

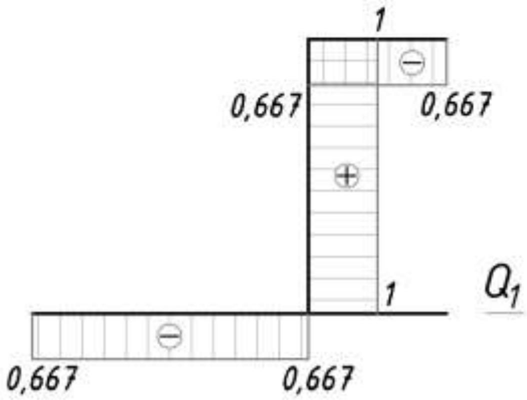
$M_2^l = -0,667 \cdot 8 = -5,333m;$

$M_3 = 0;$

$M_4^r = -0,667 \cdot 4 = -2,667m;$

$M_5^{up} = -2,667m;$

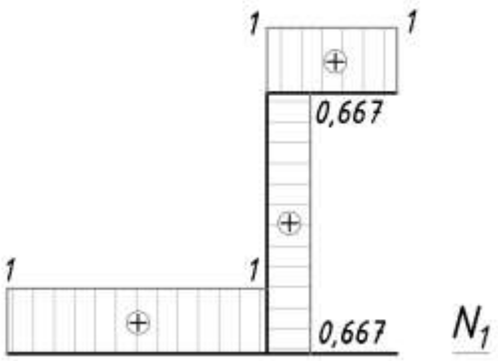
$M_6^{up} = -0,667 \cdot 4 + 1 \cdot 8 = +5,333m.$



$Q_1 = Q_2 = -0,667;$

$Q_3 = Q_4 = -0,667;$

$Q_5 = Q_6 = +1.$

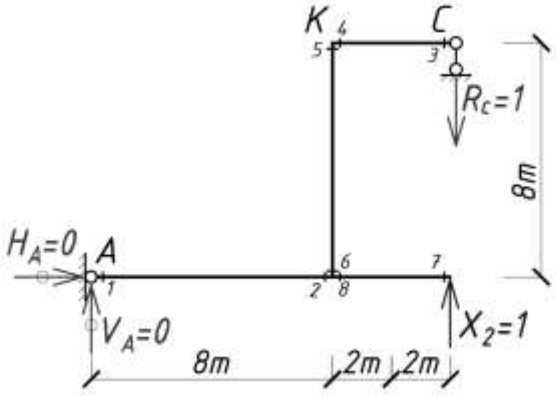


$N_1 = N_2 = +1;$

$N_3 = N_4 = +1;$

$N_5 = N_6 = +0,667.$

3.2. The second unitary state



$\sum F_x = 0:\ H_A = 0;$

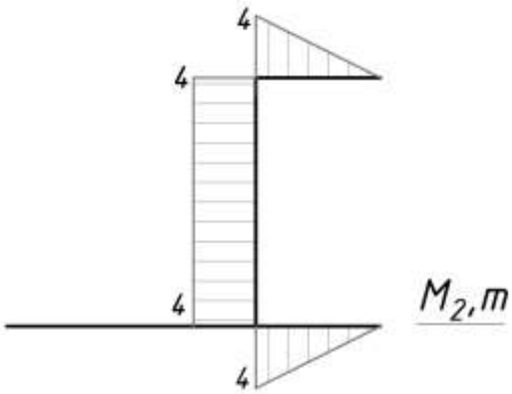
$\sum M_A = 0:\ -1 \cdot 12 + R_C \cdot 12 = 0,$

$R_C = 1;$

$\sum F_y = 0:\ V_A + 1 - 1 = 0,\ V_A = 0;$

The check:

$\sum M_K = 1 \cdot 4 - 1 \cdot 4 = 4 - 4 = 0.$



$M_1 = M_2 = 0;$

$M_3 = 0;$

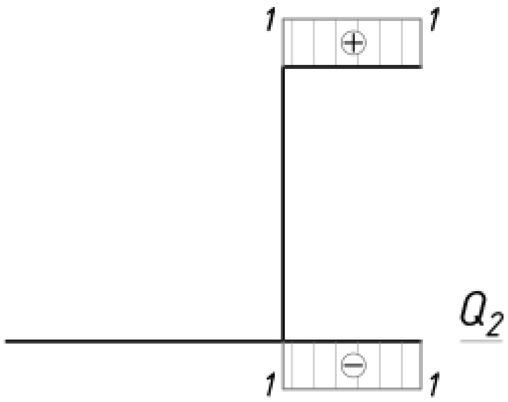
$M_4^r = +1 \cdot 4 = +4m;$

$M_5^{up} = +4m;$

$M_6^{up} = +4m;$

$M_7 = 0;$

$M_8^r = -1 \cdot 4 = -4m.$

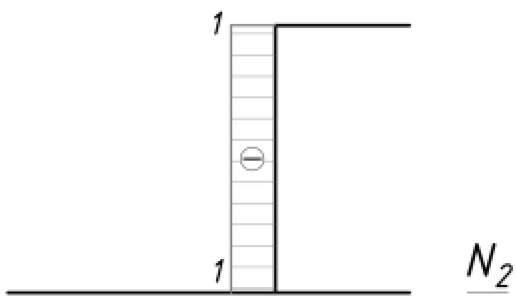


$Q_1 = Q_2 = 0;$

$Q_3 = Q_4 = +1;$

$Q_5 = Q_6 = 0;$

$Q_7 = Q_8 = -1.$



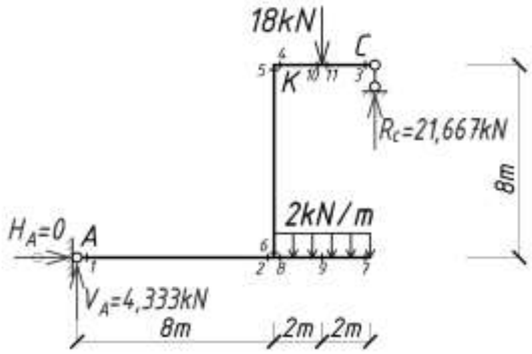
$N_1 = N_2 = 0;$

$N_3 = N_4 = 0;$

$N_5 = N_6 = -1;$

$N_7 = N_8 = 0.$

3.3. The freight state



$\sum F_x = 0: \; H_A = 0;$

$\sum M_A = 0: \; 18 \cdot 10 + 2 \cdot 4 \cdot 10 - R_C \cdot 12 = 0,$

$R_C = 21,667kN;$

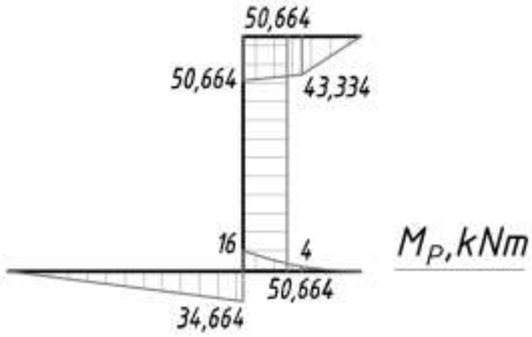
$\sum F_y = 0: \; V_A - 18 - 2 \cdot 4 + 21,667 = 0,$

$V_A = 4,333kN;$

The check:

$\sum M_K = 4,333 \cdot 8 + 18 \cdot 2 - 21,667 \cdot 4 +$

$+ 2 \cdot 4 \cdot 2 = 86,664 - 86,668 \approx 0.$



$M_1 = 0;$

$M_2^l = +4,333 \cdot 8 = +34,664kNm;$

$M_3 = 0;$

$M_{10}^r = M_{11}^r = -21,667 \cdot 2 = -43,334kNm;$

$M_4^r = -21,667 \cdot 4 + 18 \cdot 2 = -50,664kNm;$

$M_5^{up} = M_6^{up} = -50,664kNm;$

$M_7 = 0;$

$M_9^r = +2 \cdot 2 \cdot 1 = +4kNm;$

$M_8^r = +2 \cdot 4 \cdot 2 = +16kNm.$

$Q_1 = Q_2 = +4,333kN;$

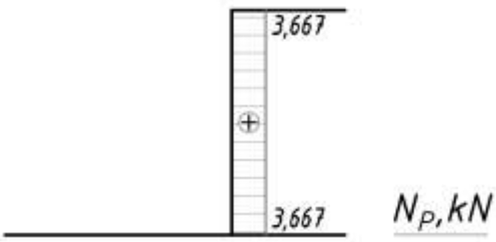
$Q_3 = Q_{11} = -21,667kN;$

$Q_{10} = Q_4 = -21,667 + 18 = -3,667kN;$

$Q_5 = Q_6 = 0;$

$Q_7 = 0;$

$Q_8 = +2 \cdot 4 = +8kN.$



$N_1 = N_2 = 0;$

$N_3 = N_{11} = N_{10} = N_4 = 0;$

$N_5 = N_6 = +21,667 - 18 = +3,667kN;$

$N_7 = N_8 = 0.$

4. Calculation of coefficients and free terms of the system of equations.

Main coefficients:

$$\delta_{11}=\sum\int\frac{M_1\cdot M_1}{EI}dx=\frac{1}{2EI}\left[\frac{1}{2}\cdot 5,333\cdot 8\cdot\frac{2}{3}\cdot 5,333\right]+\frac{1}{EI}\frac{8}{6}\Big[2,667^2+$$

$$+4\cdot\left(\frac{5,333-2,667}{2}\right)^2+5,333^2\Bigg]+\frac{1}{2EI}\left[\frac{1}{2}\cdot 2,667\cdot 4\cdot\frac{2}{3}\cdot 2,667\right]=\frac{99,545}{EI};$$

$$\delta_{22}=\sum\int\frac{M_2\cdot M_2}{EI}dx=\frac{1}{EI}\left[\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4\right]+\frac{1}{EI}\left[4\cdot 8\cdot 4\right]+\frac{1}{2EI}\left[\frac{1}{2}\cdot 4\cdot 4\cdot\frac{2}{3}\cdot 4\right]=$$

$$=\frac{160}{EI}.$$

Secondary coefficients:

$$\delta_{12}=\delta_{21}=\sum\int\frac{M_1\cdot M_2}{EI}dx=\frac{1}{EI}\left[\frac{(5,333-2,667)}{2}\cdot 8\cdot 4\right]-$$

$$-\frac{1}{2EI}\left[\frac{1}{2}\cdot 2,667\cdot 4\cdot\frac{2}{3}\cdot 4\right]=\frac{35,544}{EI}.$$

Free terms:

$$\Delta_{1P}=\sum\int\frac{M_1\cdot M_P}{EI}dx=-\frac{1}{2EI}\left[\frac{1}{2}\cdot 5,333\cdot 8\cdot\frac{2}{3}\cdot 34,664\right]-$$

$$-\frac{1}{EI}\left[\frac{(5,333-2,667)}{2}\cdot 8\cdot 50,664\right]+\frac{1}{2EI}\left[\frac{1}{2}\cdot\frac{2,667}{2}\cdot 2\cdot\frac{2}{3}\cdot 43,334\right]+$$

$$-\frac{1}{2EI}\frac{2}{6}\Bigg[2,667\cdot 50,664+4\cdot\frac{(50,664+43,334)}{2}\cdot\frac{(2,667+2,667/2)}{2}+$$

$$+\frac{2,667}{2}\cdot 43,334\Bigg]=\frac{-672,679}{EI};$$

$$\Delta_{2P}=\sum\int\frac{M_2\cdot M_P}{EI}dx=-\frac{1}{EI}\frac{4}{6}\left[4\cdot 16+4\cdot\frac{4}{2}\cdot 4\right]-\frac{1}{EI}\left[4\cdot 8\cdot 50,664\right]-$$

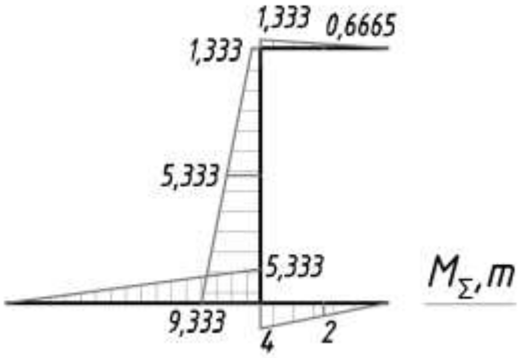
$$-\frac{1}{2EI}\left[\frac{1}{2}\cdot\frac{4}{2}\cdot 2\cdot\frac{2}{3}\cdot 43,334\right]-\frac{1}{2EI}\frac{2}{6}\Bigg[50,664\cdot 4+4\cdot\frac{(4+2)}{2}\cdot\frac{(50,664+43,334)}{2}+$$

$$+2\cdot 43,334\Big]=\frac{1856,356}{EI}.$$

5. <u>The check of coefficients of the system of equations.</u>

Depict the summary unitary bending moment diagram by using the formula:

$$M_{\Sigma} = M_1 + M_2$$



$$\delta_{1\Sigma} = \delta_{11} + \delta_{12} = \frac{99,545}{EI} + \frac{35,544}{EI} = \frac{135,089}{EI},$$

$$\delta_{1\Sigma} = \sum\int \frac{M_1 \cdot M_{\Sigma}}{EI} dx = \frac{1}{2EI}\left[\frac{1}{2} \cdot 5,333 \cdot 8 \cdot \frac{2}{3} \cdot 5,333\right] +$$

$$+\frac{1}{EI}\frac{8}{6}\left[5,333 \cdot 9,333 + 4 \cdot \frac{(5,333 - 2,667)}{2} \cdot 5,333 - 2,667 \cdot 1,333\right] +$$

$$+\frac{1}{2EI}\left[\frac{1}{2} \cdot 2,667 \cdot 4 \cdot \frac{2}{3} \cdot 1,333\right] = \frac{135,09}{EI};$$

$$\delta_{2\Sigma} = \delta_{21} + \delta_{22} = \frac{35,544}{EI} + \frac{160}{EI} = \frac{195,544}{EI},$$

$$\delta_{2\Sigma} = \sum\int \frac{M_2 \cdot M_{\Sigma}}{EI} dx = \frac{1}{EI}\left[\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 4\right] + \frac{1}{EI}\left[5,333 \cdot 8 \cdot 4\right] +$$

$$+\frac{1}{2EI}\left[\frac{1}{2} \cdot 4 \cdot 4 \cdot \frac{2}{3} \cdot 1,333\right] = \frac{195,544}{EI};$$

$$\Delta_{\Sigma P} = \Delta_{1P} + \Delta_{2P} = -\frac{672,679}{EI} - \frac{1856,356}{EI} = -\frac{2529,035}{EI},$$

$$\Delta_{\Sigma P} = \sum\int \frac{M_{\Sigma} \cdot M_P}{EI} dx = -\frac{1}{2EI}\left[\frac{1}{2} \cdot 5,333 \cdot 8 \cdot \frac{2}{3} \cdot 34,664\right] - \frac{1}{EI}\frac{4}{6}\left[4 \cdot 16 + 4 \cdot \frac{4}{2} \cdot 4\right] -$$

$$-\frac{1}{EI}\left[5,333 \cdot 8 \cdot 50,664\right] - \frac{1}{2EI}\left[\frac{1}{2} \cdot \frac{1,333}{2} \cdot 2 \cdot \frac{2}{3} \cdot 43,334\right] - \frac{1}{2EI}\frac{2}{6}\left[1,333 \cdot 50,664 +\right.$$

$$\left.+4 \cdot \frac{(1,333 + 0,6665)}{2} \cdot \frac{(50,664 + 43,334)}{2} + 0,6665 \cdot 43,343\right] = -\frac{2529,034}{EI}.$$

6. Calculation of unknowns of the system of equations.

The system of canonical equations may be written as:

$$\begin{cases} \dfrac{95,545}{EI}X_1 + \dfrac{35,544}{EI}X_2 - \dfrac{672,679}{EI} = 0, \\ \dfrac{35,544}{EI}X_1 + \dfrac{160}{EI}X_2 - \dfrac{1856,356}{EI} = 0; \end{cases}$$

$$\begin{cases} 95,545 \cdot X_1 + 35,544 \cdot X_2 = 672,679, \\ 35,544 \cdot X_1 + 160 \cdot X_2 = 1856,356. \end{cases}$$

For solving of the system use the method of substitution. From second equation can write:

$$X_1 = \frac{1856,356 - 160 \cdot X_2}{35,544} = 52,227 - 4,501 \cdot X_2 .$$

Substitute in the first equation:

$$95,545 \cdot (52,227 - 4,501 \cdot X_2) + 35,544 \cdot X_2 = 672,679 ,$$

$$5198,937 - 448,052 \cdot X_2 + 35,544 \cdot X_2 = 672,679 ,$$

$$412,508 \cdot X_2 = 4526,258 ,$$

$$X_2 = 10,973 kN ,$$

$$X_1 = 52,227 - 4,501 \cdot 10,973 = 2,838 kN .$$

For checking substitute in the summary equation:

$$135,089 \cdot X_1 + 195,544 \cdot X_2 = 2529,035 ,$$

$$135,089 \cdot 2,838 + 195,544 \cdot 10,973 - 2529,035 = 0 ,$$

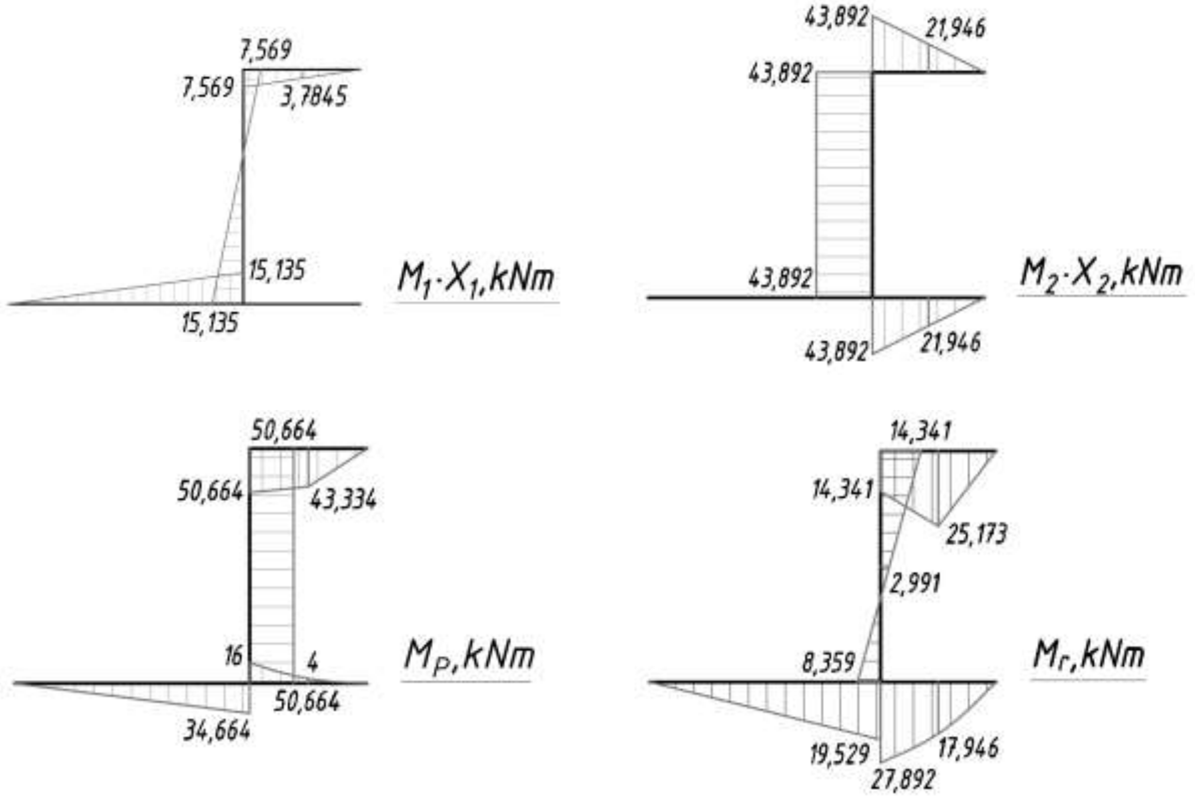
$$0,052 \approx 0 .$$

Отже, $X_1 = 2,838 kN$, $X_2 = 10,973 kN$.

7. <u>Depiction of real internal forces diagrams.</u>

Real diagrams of internal forces depict by using the method of superposition:

$$M_r = M_1 \cdot X_1 + M_2 \cdot X_2 + M_P \ .$$



The kinematic check:

$$\Delta_{\Sigma r} = \sum \int \frac{M_\Sigma \cdot M_r}{EI} dx = -\frac{1}{2EI}\left[\frac{1}{2} \cdot 5,333 \cdot 8 \cdot \frac{2}{3} \cdot 19,529\right] + \frac{1}{EI}\frac{4}{6}\left[4 \cdot 27,892 + \right.$$

$$\left. +4 \cdot 2 \cdot 17,946\right] + \frac{1}{EI}\frac{8}{6}\left[9,333 \cdot 8,359 - 4 \cdot 5,333 \cdot 2,991 - 1,333 \cdot 14,341\right] -$$

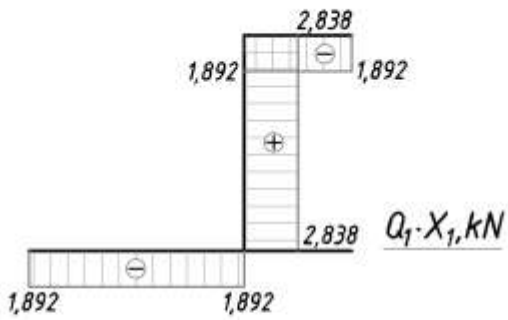
$$-\frac{1}{2EI}\left[\frac{1}{2} \cdot 0,6665 \cdot 2 \cdot \frac{2}{3} \cdot 25,173\right] - \frac{1}{2EI}\frac{2}{6}\left[1,333 \cdot 14,341 + 4 \cdot \frac{(1,333 + 0,6665)}{2} \times \right.$$

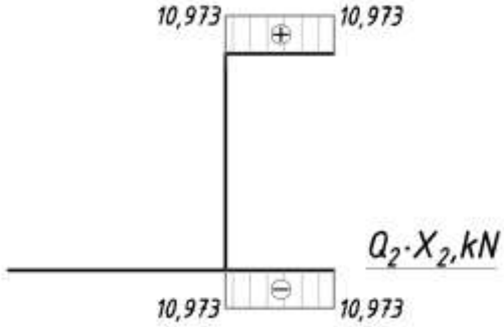
$$\left. \times \frac{(14,341 + 25,173)}{2} + 0,6665 \cdot 25,173\right] = \frac{1}{EI}\left[170,091 - 170,148\right] = -\frac{0,057}{EI} \approx 0$$

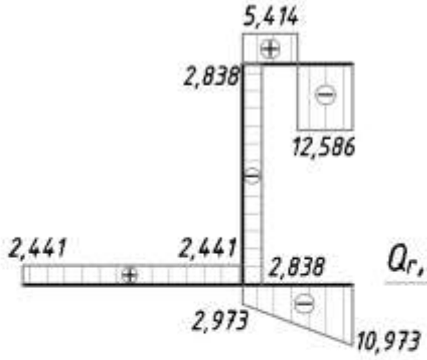
The relative fault is:

$$\varepsilon = \frac{0,057}{170,091} \cdot 100\% = 0,0335\% \prec 2\% .$$

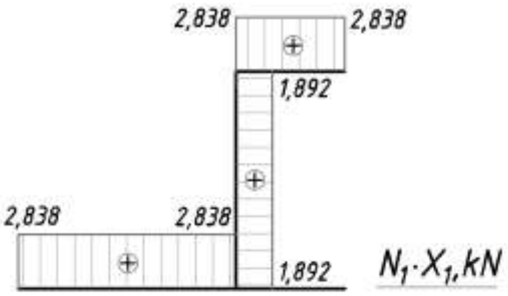
$$Q_r = Q_1 \cdot X_1 + Q_2 \cdot X_2 + Q_P \ .$$

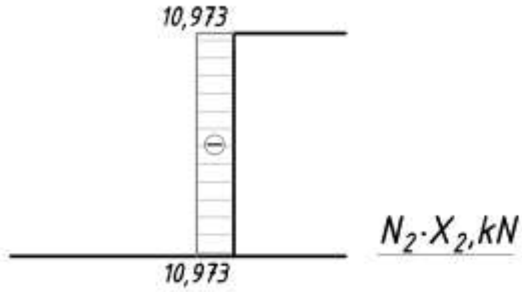


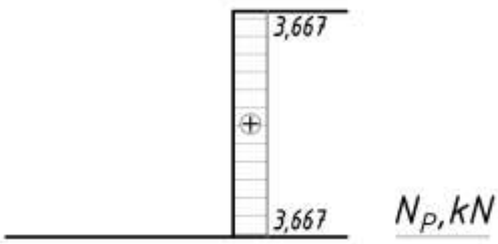


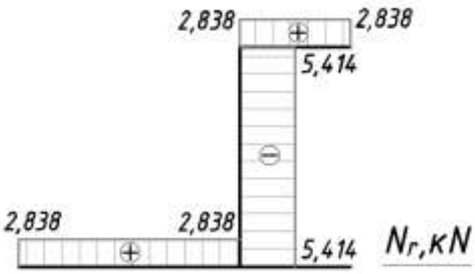


$$N_r = N_1 \cdot X_1 + N_2 \cdot X_2 + N_P \ .$$



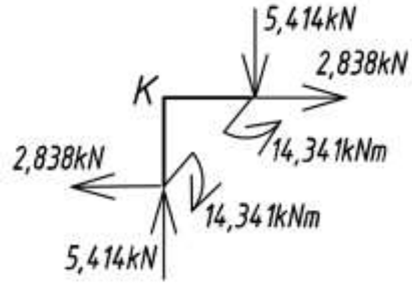






7. <u>Checks of real internal forces diagrams</u>.

The check of equilibrium conditions of free bodies of joints of the frame:

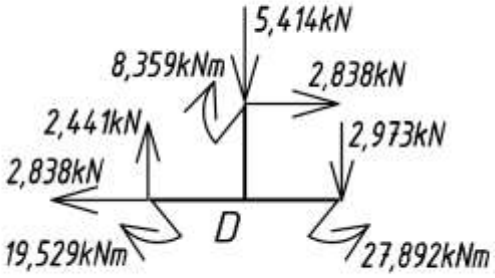


The free body of joint K:

$\sum F_x = -2{,}838 + 2{,}838 = 0$,

$\sum F_y = 5{,}414 - 5{,}414 = 0$,

$\sum M_K = 14{,}341 - 14{,}341 = 0$.



The free body of joint D:

$\sum F_x = -2{,}838 + 2{,}838 = 0$,

$\sum F_y = 2{,}441 + 2{,}973 - 5{,}414 = 0$,

$\sum M_K = 19{,}529 + 8{,}359 - 27{,}892 = 0$.

The check of the relationship between M and Q diagrams:

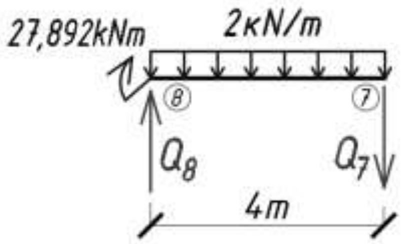
$$Q_{1-2} = -\frac{19{,}529 - 0}{8} = -2{,}5441 kN ,$$

$$Q_{5-6} = +\frac{14{,}529341 - (-8{,}359)}{8} = +2{,}838 kN ,$$

$$Q_{3-11} = -\frac{25{,}173 - 0}{2} = +12{,}5865 kN \approx 12{,}586 kN ,$$

$$Q_{4-10} = +\frac{25{,}173 - 14{,}341}{2} = +5{,}416 kN \approx 5{,}414 kN ,$$

Q_{7-8}:



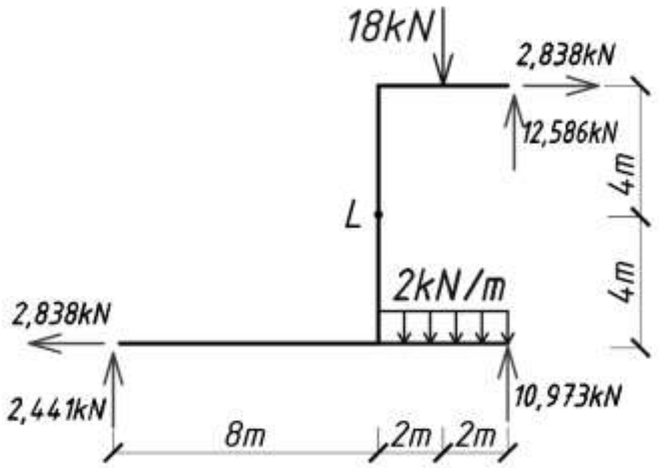
$\sum M_8 = 0: \quad 27{,}892 + 2 \cdot 4 \cdot 2 + Q_7 \cdot 4 = 0$;

$Q_7 = -10{,}973 kN$.

$\sum M_7 = 0: \quad 27{,}892 - 2 \cdot 4 \cdot 2 + Q_8 \cdot 4 = 0$;

$Q_8 = +2{,}973 kN$.

8. <u>The check of equilibrium conditions of the whole system.</u>



$\sum F_x = -2{,}8388 + 2{,}838 = 0$,

$\sum F_y = 2{,}441 - 2 \cdot 4 - 18 + 10{,}973 + 12{,}586 = 26 - 26 = 0$,

$\sum M_L = 2{,}838 \cdot 4 + 2{,}441 \cdot 8 - 2 \cdot 4 \cdot 2 - 10{,}973 \cdot 4 + 18 \cdot 2 - 12{,}586 \cdot 4 + 2{,}838 \cdot 4 =$
$= 94{,}232 - 94{,}197236 = -0{,}004 \approx 0$.

All checks are satisfied therefore values of internal forces of the original structure are calculated correctly.

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Educational and methodical edition

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Methodical instructions and individual tasks
for performing of the calculation-graphic work
for students of the specialty 192 «Construction and Civil Engineering»
of the education program «Urban construction and governance»

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